PARTIAL DIFFERENTIAL EQUATION

Previous year Questions from 1992 To 2017

Syllabus

Family of surfaces in three dimensions and formulation of partial differential equations; Solution of quasilinear partial differential equations of the first order, Cauchy's method of characteristics; Linear partial differential equations of the second order with constant coefficients, canonical form; Equation of a vibrating string, heat equation, Laplace equation and their solutions.

** Note: Syllabus was revised in 1990’s and 2001 & 2008 **
2017

1. Solve \((D^2 - 2DD' + D'^2)z = e^{x^2+y} + x^3 + \sin 2x\), where \(D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}, D^2 = \frac{\partial^2}{\partial x^2}, D'^2 = \frac{\partial^2}{\partial y^2}\). [10 Marks]

2. Find a complete integral of the partial differential equation \(2(q\bar{p} + \bar{q}x) + x^2 + y^2 = 0\). [15 Marks]

3. Reduce the equation \(y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}\) to canonical form and hence solve it. [15 Marks]

4. Given the one-dimensional wave equation \(\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, t > 0\), where \(c^2 = \frac{T}{m}\), \(T\) is the constant tension in the string and \(m\) is the mass per unit length of the string.
   (i) Find the appropriate solution of the above wave equation.
   (ii) Find also the solution under the conditions \(y(0,t), y(l,t) = 0\) for all \(t\) and \(\frac{\partial y}{\partial t}|_{t=0} = 0, y(x,0) = a \sin \frac{\pi x}{l}, 0 < x < l, a > 0\) [20 Marks]

2016

5. Find the general equation of surfaces orthogonal to the family of spheres given by \(x^2 + y^2 + z^2 = cz\) [10 Marks]

6. Find the general integral of the partial differential equation \((y+zx)p - (x+yz)q = x^2 - y^2\) [10 Marks]

7. Determine the characteristics of the equation \(z = p^2 - q^2\) and find the integral surface which passes through the parabola \(4z + x^2 = 0, y = 0\) [15 Marks]

8. Solve the particle differential equation \(\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} \frac{\partial^3 z}{\partial x \partial y^2} + 2 \frac{\partial^3 z}{\partial y^3} = e^{x+y}\) [15 Marks]

9. Find the temperature \(u(x,t)\) in a bar of silver of length 10cm and constant cross section of area 1cm². Let density \(p = 10.6 \text{ g/cm}^3\), thermal conductivity \(k = 1.04 / (\text{cm sec °C})\) and specific heat \(\sigma = 0.056/\text{g°C}\) the bar is perfectly isolated laterally with ends kept at 0°C and initial temperature \(f(x) = \sin(0.1 \pi x)°C\) note that \(u(x,t)\) follows the heat equation \(ut = c^2u_{xx}\) where \(c^2 = k / (p\sigma)\) [20 Marks]
10. Solve the partial differential equation: \((y^2+z^2-x^2)p-2xyz+2xz=0\) where \(p = \frac{\partial z}{\partial x}\) and \(q = \frac{\partial z}{\partial y}\)  
\[10 \text{ Marks}\]

11. Solve: \((D^2+DD'-2D^2)u = e^{xy}\), where \(D = \frac{\partial}{\partial x}\) and \(D' = \frac{\partial}{\partial y}\)  
\[10 \text{ Marks}\]

12. Solve for the general solution \(pcos(x+y)+qsin(x+y)z\), where \(p = \frac{\partial z}{\partial x}\) and \(q = \frac{\partial z}{\partial y}\)  
\[15 \text{ Marks}\]

13. Find the solution of the initial boundary value problem  
\[u_t-u_{xx}+u=0, \quad 0 < x < l, \ t > 0\]  
\[u(0,t) = u(l,t), \quad t \geq 0\]  
\[u(x,0) = x(l-x), \quad 0 < x < l\]  
\[15 \text{ Marks}\]

14. Reduce the second order partial differential equation  
\[x^2 = \frac{\partial^2 y}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0\]  
into canonical form. Hence, find its general solution  
\[15 \text{ Marks}\]

15. Solve the partial differential equation \((2D^2\vec{5}DD'+2D')z=24(y-x)\)  
\[10 \text{ Marks}\]

16. Reduce the equation \(\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}\) to canonical form.  
\[15 \text{ Marks}\]

17. Find the deflection of a vibrating string (length = \(\pi\), ends fixed, \(\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}\)) corresponding to zero initial velocity and initial deflection.  
\(f(x) = k(sinx-sin2x)\)  
\[15 \text{ Marks}\]

18. Solve \(\frac{\partial^3 u}{\partial t^2} = \frac{\partial^3 u}{\partial x^3}, 0 < x < 1, t > 0\), given that  
\(i)\ \ u(x,0)=0, \ 0 \leq x \leq 1;\)  
\(ii)\ \ u_t (x,0)=x^2, \ 0 \leq x \leq 1\)  
\(iii)\ \ u(0,t)=u(1,t)=0, \ \text{for all}\ t\)  
\[15 \text{ Marks}\]

19. From a partial differential equation by eliminating the arbitrary functions \(f\) and \(g\) from  
\(z = yf(x)+xg(y)\)  
\[10 \text{ Marks}\]

20. Reduce the equation \(y \frac{\partial^2 z}{\partial x^2} + (x+y) \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0\) to its canonical form when \(x \neq y\)  
\[10 \text{ Marks}\]
21. Solve \((D^2+DD'-6D^2)z=x^2\sin(x+y)\) where \(D\) and \(D'\) denote \(\frac{\partial}{\partial x}\) and \(\frac{\partial}{\partial y}\) \([15\text{ Marks}]\)

22. Find the surface which intersects the surfaces of the system \(z(x+y)=C(3z+l)\), \((C\) being a constant) orthogonally and which passes through the circle \(x^2+y^2=1\), \(z=1\) \([15\text{ Marks}]\)

23. A tightly stretched string with fixed end points \(x=0\) and \(x=l\) is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity \(\lambda x(l-x)\), find the displacement of the string at any distance \(x\) from one end at any time \(t\) \([20\text{ Marks}]\)

2012

24. Solve partial differential equation \((D-2D')(D-D)^2z=e^{xy}\) \([12\text{ Marks}]\)

25. Solve partial differential equation \(px+qy=3z\) \([20\text{ Marks}]\)

26. A string of length \(l\) is fixed at its ends. The string from the mid-point is pulled up to a height \(k\) and then released from rest. Find the deflection \(y(x,t)\) of the vibrating string. \([20\text{ Marks}]\)

27. The edge \(r=a\) of a circular plate is kept at temperature \(f(\theta)\). The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. \([20\text{ Marks}]\)

2011

28. Solve the PDE \((D^2+D-2+3D')z=e^{x^2}z\) \([12\text{ Marks}]\)

29. Solve the PDE \(x+2z\frac{\partial z}{\partial x}+(4xz-y)\frac{\partial z}{\partial y}=2x^2+y\) \([12\text{ Marks}]\)

30. Find the surface satisfying \(\frac{\partial^2 z}{\partial x^2}=6x+2\) and touching \(z=x^3+y^3\) along its section by the plane \(x+y+z=0\). \([20\text{ Marks}]\)

31. Solve \(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0\), \(0≤x≤a\), \(0≤y≤b\) satisfying the boundary conditions \(u(0,y)=0\), \(u(x,0)=0\), \(u(x,b)=0\), \(\frac{\partial u}{\partial x}(a,y)=T\sin\frac{\pi y}{a}\) \([20\text{ Marks}]\)

32. Obtain temperature distribution \(y(x,t)\) in a uniform bar of unit length whose one end is kept at \(10^\circ\) and the other end is insulated. Also it is given that \(y(x,0)=1-x\), \(0<x<1\). \([20\text{ Marks}]\)

2010

33. Solve the PDE \((D^2-D')(D-2D')Z=e^{2xy}+xy\) \([12\text{ Marks}]\)

34. Find the surface satisfying the PDE \((D^2-2DD'+3D^2)Z=0\) and the conditions that \(bZ=y^2\) when \(x=0\) and \(aZ=x^2\) when \(y=0\) \([12\text{ Marks}]\)

35. Solve the following partial differential equation \(zp+yq=x\)
\(x_0(s)=s\), \(y_0(s)=1\), \(z_0(s)=2s\)
by the method of characteristics. \([20\text{ Marks}]\)

36. Reduce the following 2nd order partial differential equation into canonical form and
37. Solve the following heat equation
\[ u_t - u_{xx} = 0, \quad 0 < x < 2, \quad t > 0 \]
\[ u(0, t) = u(2, t) = 0, \quad t > 0 \]
\[ u(x, 0) = x(2-x), \quad 0 \leq x \leq 2 \]  [20 Marks]

2009

38. Show that the differential equation of all cones which have their vertex at the origin is
\[ px + qy = z. \]
Verify that this equation is satisfied by the surface \( yz + zx + xy = 0 \).  [12 Marks]

39. (i) Form the partial differential equation by elimination the arbitrary function \( f \) given by:
\[ f(x^2 + y^2, z-xy) = 0 \]
(ii) Find the integral surface of:
\[ x^2 p + y^2 p + z^2 = 0 \]
which passes through the curve:
\[ xy = x + y, \quad z = 1 \]  [20 Marks]

40. Find the characteristics of:
\[ y^2 r - x^2 t = 0 \]
where \( r \) and \( t \) have their usual meanings.  [15 Marks]

41. Solve:
\[ (D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2) \sin xy - \cos xy \]
where \( D \) and \( D' \) represent \( \frac{\partial}{\partial x} \) and \( \frac{\partial}{\partial y} \).  [15 Marks]

42. A tightly stretched string has its ends fixed at \( x = 0 \) and \( x = 1 \). At time \( t = 0 \), the string is given a shape defined by \( f(x) = \mu x(1-x) \), where \( \mu \) is a constant, and then released. Find the displacement of any point \( x \) of the string at time \( t > 0 \).  [20 Marks]

2008

43. Find the general solution of the partial differential equation:
\[ (2xy-1)p + (z-2x^2)q = 2(x-zy) \]
and also find the particular solution which passes through the lines \( x = 1, \ y = 0 \)  [12 Marks]

44. Find the general solution of the partial differential equation:
\[ (D^2 + DD' - 6D'^2)z = y \cos x, \]
where \( D = \frac{\partial}{\partial x}, \ D' = \frac{\partial}{\partial y} \)  [12 Marks]

45. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines \( x = 0, \ x = a, \ y = 0 \) and \( y = b \). The edges and \( x = 0 \), and \( x = a \) and \( y = 0 \) are kept at temperature zero while the edge \( y = b \) is kept at \( 100^\circ C \).  [30 Marks]

46. Find complete and singular integrals of \( 2xz - px^2 - 2qxy + pq = 0 \) using Charpit’s method.  [15 Marks]

47. Reduce \( \frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2} \) canonical form.  [15 Marks]

2007

48. (i) Form a partial differential equation by eliminating the function \( f \) from:
\[ z = y^2 + 2f \left( \frac{1}{x} + \log y \right) \]
(ii) Solve \( 2x - px^2 - 2qxy + pq = 0 \) \[12 \text{ Marks}\]

49. Transform the equation \( yzx - xzy = 0 \) into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. \[12 \text{ Marks}\]

50. Solve \( u_x + u_y = 0 \) in \( D \) where \( D = \{ (x, y): 0 < x < a, 0 < y < b \} \) is a rectangle in a plane with the boundary conditions:

\[
\begin{align*}
  u(x, 0) &= 0, \quad u(x, b) = 0, \quad 0 \leq x \leq a \\
  u(0, y) &= g(y), \quad u_x(a, y) = h(y), \quad 0 \leq y \leq b 
\end{align*}
\]

\[30 \text{ Marks}\]

51. Solve the equation \( \frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \) by separation of variables method subject to the conditions:

\( u(0, t) = 0 = u(l, t), \) for all \( t \) and \( u(x, 0) = f(x) \) for all \( x \) in \([0, l]\) \[30 \text{ Marks}\]

2006

52. Solve: \( px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3) \) \[12 \text{ Marks}\]

53. Solve:

\[
\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^2 z}{\partial x \partial y^2} = 2 \sin(3x + 2y) \]

\[12 \text{ Marks}\]

54. The deflection of vibrating string of length \( l \), is governed by the partial differential equation \( u_{tt} = C^2 u_{xx} \). The ends of the string are fixed at \( x = 0 \) and \( l \). The initial velocity is zero. The initial displacement is given by

\[
u(x, 0) = \begin{cases} 
  \frac{x}{l}, & 0 < x < \frac{l}{2} \\
  \frac{1}{l}(l - x), & \frac{l}{2} < x < l.
\end{cases}
\]

Find the deflection of the string at any instant of time. \[30 \text{ Marks}\]

55. Find the surface passing through the parabolas \( z = 0, y^2 = 4ax \) and \( z = 1, y^2 = -4ax \) and satisfying the equation \( x \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial z}{\partial x} = 0 \) \[15 \text{ Marks}\]

56. Solve the equation \( p^2x + q^2y = z, \quad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y} \) \[15 \text{ Marks}\]

2005

57. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. \[12 \text{ Marks}\]

58. Find the particular integral of \( x(y - z)p = y(z - x)q = z(x - y) \) which represents a surface passing through \( x = y = z \) \[12 \text{ Marks}\]

59. The ends \( A \) and \( B \) of a rod 20cm long have the temperature at 30°C and 80°C until steady state prevails. The temperatures of ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the rod at time \( t \). \[30 \text{ Marks}\]

60. Obtain the general solution of \((D - 3D')^2 z = 2e^{2z} \sin(y + 3x)\) where \( D = \frac{\partial}{\partial x} \) and \( D' = \frac{\partial}{\partial y} \) \[15 \text{ Marks}\]
2004

61. Find the integral surface of the following partial differential equation:
\[ x(y^2+z)p−y(x^2+z)q=(x^2−y^2)z \]  
[12 Marks]

62. Find the complete integral of the partial differential equation \((p^2+q^2)x=pz\) and deduce the solution which passes through the curve \(x=0, z^3=4y\).  
[12 Marks]

63. Solve the partial differential equation:
\[ \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = (y-1)e^x \]  
[15 Marks]

64. A uniform string of length \(l\), held tightly between \(x=0\) and \(x=l\) with no initial displacement, is struck at \(x=a, 0<a<l\), with velocity \(v_0\). Find the displacement of the string at any time \(t>0\).  
[15 Marks]

65. Using Charpit’s method, find the complete solution of the partial differential equation \(p^2x+q^2y=z\).  
[15 Marks]

2003

66. Find the general solution of \[ \frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y + \cos(2x + 3y) \]  
[12 Marks]

67. Show that the differential equations of all cones which have their vertex at the origin are \(px+qy=z\). Verify that \(yz+zx+xy=0\) is a surface satisfying the above equation.  
[12 Marks]

68. Solve \[ \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} = xy + e^{x^2y} \]  
[15 Marks]

69. Solve the equation \(p^2q^2−2px−2qy+2xy=0\) using Charpit’s method. Also find the singular solution of the equation, if it exists.  
[15 Marks]

70. Find the deflection \(u(x,t)\) of a vibrating string, stretched between fixed points \((0,0)\) and \((3l,0)\), corresponding to zero initial velocity and following initial deflection:

\[
\begin{align*}
  f(x) &= \frac{hx}{l} , & \text{when } 0 < x < 1 \\
  f(x) &= \frac{h(3l−2x)}{l} , & \text{when } \frac{l}{2} < x < 2l \\
  f(x) &= \frac{h(x−3l)}{l} , & \text{when } 2l \leq x \leq 3l 
\end{align*}
\]

Where \(h\) is a constant.  
[15 Marks]

2002

71. Find two complete integrals of the partial differential equation \(x^2p^2+y^2q^2−4=0\)  
[12 Marks]

72. Find the solution of the equation \(z = \frac{1}{2}(p^2 + q^2) + (p − x)(q − y)\)  
[12 Marks]
73. Frame the partial differential equation by eliminating the arbitrary constants a and b from \( \log(az-1) = x + ay + b \)  

[10 Marks]

74. Find the characteristics strips of the equation \( xp+yq-pq=0 \) and then find the equation of the integral surface through the curve \( z = \frac{x}{2}, y = 0 \)  

[20 Marks]

75. Solve: \( \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0 \)  

\( u(0,t) = u(l,t) = 0 \)  

\( u(x,0) = x(l-x), 0 \leq x \leq t. \)  

[20 Marks]

76. Find the complete integral partial differential equation \( 2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2+y^2) \)  

[12 Marks]

77. Find the general integral of the equation  

\[ \{mx(x+y) - nz^2\} \frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\} \frac{\partial z}{\partial y} = (lx-my)z \]  

[12 Marks]

78. Prove that for the equation \( z + px + qy - 1 - pqx^2y^2 = 0 \) the characteristic strips are given by  

\[ x(t) = \frac{1}{B + Ce^{-t}}, \quad y(t) = \frac{1}{A + De^{-t}}, \quad z(t) = E - ACBD e^{-t} \]  

\( p(t) = A\left( B + Ce^{-t}\right)^2, q(t) = B\left( A + De^{-t}\right)^2 \) where \( A, B, C, D \) and \( E \) are arbitrary constants. 

Hence find the values of these arbitrary constants if the integral surface passes through the line \( z = 0, x = y \)  

[30 Marks]

79. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by \( x(x^2+y^2+z^2) = C \)  

[10 Marks]

80. Solve the equation  

\[ 2x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = x^2 y^4 \]  

by reducing it to the equation with constant coefficients. 

[20 Marks]

2001

81. Solve: \( pq = xmy^n z^m \)  

[12 Marks]

82. Prove that if \( x_j^3 + x_k^3 + x_j^3 = 1 \) when \( z = 0 \), the solution of the equation  

\( (S-x_j)P_1 + (S-x_k)P_2 + (S-x_l)P_3 = S-z \) can be given in the form  

\[ S^4 \{x_j(z)^3 + (x_k-z)^3 + (x_l-z)^3\} = (x_j + x_k + x_l - 3z) \]  

\( \text{where} \ S = x_j + x_k + x_l + z \) and \( P_i = \frac{\partial z}{\partial x_i}, i = 1, 2, 3. \)  

[12 Marks]

83. Solve by Charpit’s method the equation  

\[ p^2 x(x-1) + 2pqy + q^2 y(y-1) - 2pxz - 2qyz + z^2 = 0 \]  

[15 Marks]

84. Solve: \( (D^2 - DD' - 2D^2)z = 2x + 3y + e^{x+y} \).  

[15 Marks]
85. A tightly stretched string with fixed end points \(x=0, x=l\) is initially at rest in equilibrium position. If it is set vibrating by giving each point \(x\) of it a velocity \(kx(l-x)\), obtain at time \(t\) the displacement \(y\) at a distance \(x\) from the end \(x=0\) \[30\text{ Marks}\]

1999

86. Verify that the differential equation \((y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0\) is integrable and find its primitive. \[12\text{ Marks}\]

87. Find the surface which intersects the surfaces of the system \(z(x+y)=c(3z+1), c\) is constant, orthogonally and which passes through the circle \(x^2+y^2=1, z=1\) \[12\text{ Marks}\]

88. Find the characteristics of the equation \(pq=z\), and determine the integral surface which passes through the passes through the parabola \(x = 0, y^2=z\) \[15\text{ Marks}\]

89. Use Charpit’s method to find a complete integral to \(p^2+q^2-2px-2qy+1=0\) \[15\text{ Marks}\]

90. Find the solution of the equation \(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-x}\cos y\) which \(\to 0\) as \(x \to \infty\) and has the value \(\cos y\) when \(x = 0\) \[15\text{ Marks}\]

91. One end of a string \((x=0)\) is fixed, and the point \(x=a\) is made to oscillate, so that at time \(t\) the displacement is \(g(t)\). Show that the displacement \(u(x,t)\) of the point \(x\) at time \(t\) is given by \[15\text{ Marks}\]

1998

92. Find the differential equation of the set of all right circular cones whose axes coincide with the \(z\)-axis \[12\text{ Marks}\]

93. Form the differential equation by eliminating \(a, b\) and \(c\) from \(z = a(x+y) + b(x-y) + abt + c\) \[12\text{ Marks}\]

94. Solve \(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = xyz\) \[15\text{ Marks}\]

95. Find the integral surface of the linear partial differential equation \(x(y^2 + z) \frac{\partial z}{\partial x} - y(x^2 + z) \frac{\partial z}{\partial y} = y(x^2 - y^2)z\) \[15\text{ Marks}\]

96. Use Charpit's method to find a complete integral of \(\left[2x \left( z \frac{\partial z}{\partial y} \right)^2 + 1 \right] = z \frac{\partial z}{\partial x}\) \[15\text{ Marks}\]

97. Find a real function \(V(x,y)\) which reduces to zero when \(y=0\) and satisfies the equation \(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi \left(x^2 + y^2\right)\) \[20\text{ Marks}\]
98. Apply Jacobi’s method to find a complete integral of the equation

$$2 \frac{\partial z}{\partial x_1} x_1 x_3 + 3 \frac{\partial z}{\partial x_2} x_2^3 + \left( \frac{\partial z}{\partial x_2} \right)^2 \frac{\partial z}{\partial x_3} = 0$$  [20 Marks]

1997

99. (i) Find the differential equation of all surfaces of revolution having z-axis as the axis of rotation.
(ii) Form the differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$  [20 Marks]

100. Find the equation of surfaces satisfying $4yzp+q+2y=0$ and passing through $y^2+z^2=1, x+z=2$  [15 Marks]
101. Solve: $(y+z)p+(z+x)q=x+y$  [12 Marks]
102. Use Charpit’s method to find complete integral of $z'(p^2z'+q')=1$  [10 Marks]
103. Solve: $(D_x^3-D_y^3)z=x'y'$  [15 Marks]
104. Apply Jacobi’s method to find complete integral of $p_1^3+p_2^3+p=1$. Here

$$p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}, p_3 = \frac{\partial z}{\partial x_3} \text{ and } z \text{ is a function of } x_1 x_2 x_3.$$  [20 Marks]

1996

105. (i) differential equation of all spheres of radius $\lambda$ having their center in xy-plane
(ii) Form differential equation by eliminating f and g from $z=f(x^2-y)+g(x^2+y)$  [20 Marks]
106. Solve: $z'(p^2+q^2+1)=C^2$  [10 Marks]
107. Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ passing through the curve $xz=a^3, y=0$  [15 Marks]
108. Apply Charpit’s method to find the complete integral of $z=px+ay+p^2+q^2$.  [15 Marks]
109. Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$  [15 Marks]
110. Find a surface passing through the lines $z=x=0$ and $z=1=x-y=0$ satisfying

$$\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 4 \frac{\partial^2 z}{\partial y^2} = 0$$  [15 Marks]

1995

111. In the context of a partial differential equation of the first order in there independent variables, define and illustrate the terms:
(i) The complete the terms:
(ii) The singular integral  [20 Marks]
112. Find the general integral of $(y+z+w)\frac{\partial w}{\partial x}+(z+x+w)\frac{\partial w}{\partial y}+(x+y+w)\frac{\partial w}{\partial z}=x+y+z$  [15 Marks]
113. Obtain the differential equation of the surfaces which are the envelopes of a one parameter family of planes. [15 Marks]

114. Explain in detail the Charpit’s method of solving the nonlinear partial differential equation

\[ f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0 \]  

[15 Marks]

115. Solve \( \frac{\partial z}{\partial x_1} \frac{\partial z}{\partial x_2} \frac{\partial z}{\partial x_3} = x_1 x_2 x_3 \) [15 Marks]

116. Solve \( (D_x^3 - 7D_x D_y^2 - 6D_y^3)z = \sin(x + 2y) + e^{3x+y} \) [15 Marks]

1994

117. Find the differential equation of the family of all cones with vertex at \((2, -3, 1)\) [15 Marks]

118. Find the integral surface of \( x^2 p + y^2 q + z^2 = 0 \), which passes through the hyperbola \( xy = x + y, z = 1 \) [20 Marks]

1993

119. Obtain a Complete Solution of \( pq = x^n y^n z^m \) [20 Marks]

120. Use the Charpit’s method to solve \( 16p^2 z^2 + 9q^2 z^2 + 4z^2 - 4 = 0 \). Interpret geometrically the complete solution and mention the singular solution. [20 Marks]

121. Solve \( (D_x^2 + 3DD_y')z = x + y \), by expanding the particular integral in ascending powers of \( D \), as well as in ascending powers of \( D' \). [20 Marks]

1992

122. Find a surface satisfying \( (D_x^2 + DD_y)z = 0 \) and touching the elliptic paraboloid \( z = 4x^2 + y^2 \) along its section by the plane \( y = 2x + 1 \). [20 Marks]

123. Find the surface whose tangent planes cut off an intercept of constant length \( R \) from the axis of \( z \). [20 Marks]

124. Solve \( (x^2 + 3xy^2)p + (y^2 + 3x^2)yq = 2(x^2 + y^2)z \) [20 Marks]

125. Find the integral surface of the partial differential equation \( (x - y)p + (y - z)q = z \) through the circle \( z = 1, x^2 + y^2 = 1 \) [20 Marks]

126. Using Charpit’s method find the complete integral of \( 2xz - px^2 - 2qxy + pq = 0 \) [15 Marks]

127. Solve \( r - s + 2q - z = x^2 y^2 \) [15 Marks]

128. Find the general solution of \( x^2 r - y^2 t + xp - yq = \text{log} x \) [20 Marks]

129. Solve:

\[
(2x^2 - y^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + 2z^2 - yz - zx - 2xy)
\]

[20 Marks]

130. Find the complete integral of \( (y - x)(qy - px) = (p - q)^2 \) [20 Marks]

131. Use Charpit’s method to solve \( px + qy = z \sqrt{1 + pq} \) [20 Marks]

132. Find the surface passing through the parabolas \( z = 0, y^2 = 4ax; z = 1, y^2 = -4ax \) and satisfying the differential equation \( xr + 2p = 0 \) [20 Marks]

133. Solve: \( r + s - 6t = y \cos x \) [20 Marks]
134. Solve: \[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y) + e^y
\] [20 Marks]