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MATHEMATICS

MODERN ALGEBRA

Previous year Questions from **1992 To 2017**

Syllabus

Groups, subgroups, cyclic groups, cosets, Lagrange's Theorem, normal subgroups, quotient groups, homomorphism of groups, basic isomorphism theorems, permutation groups, Cayley's theorem.

Rings, subrings and ideals, homomorphisms of rings; Integral domains, principal ideal domains, Euclidean domains and unique factorization domains; Fields, quotient fields.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

2017

1. Let F be a field and F[X] denote the ring of polynomials over F in a single variable X. For f(X), g(X) $\in$ F[X] with g(X) $\neq$ 0, show that there exist q(X), f(X)$\in$F[X] such that degree (r(X)) < degree (g(X)) and f(X)=q(X).g(X)+r(X). **(20 marks)**

2. Show that the groups $Z_5 \times Z_7$ *and* Z_{35} are isomorphic. **(15 marks)**

2016

3. Let K be a field and $K[X]$ be the ring of polynomials over K in a single variable X for a polynomial $f \in K$ [x] Let (f) denote the ideal in $K[X]$ generated by f. show that (f) is a maximal ideal in $K[X]$ if and only if is an irreducible polynomial over K. **(10 marks)**

4. Let p be prime number and Z_p denote the additive group of integers modulo p. Show that every non-zero element Z_p of generates Z_p **(15 marks)**

5. Let K be an extension of field F prove that the element of K which are algebric over F form a subfield of K further if $F \subset K \subset L$ fare fields L is algebric over K and K is algebraic over F then prove that L is algebraic over F. **(20 marks)**

6. Show that every algebraically closed field is infinite. **(15 marks)**

2015

7. (i) How many generators are there of the cyclic group G of order 8? Explain. **(5 marks)**

   (ii) Taking a group {e,a,b,c} of order 4, where e is the identity, construct compostition tables showing that one is cyclic while the other is not **(5 marks)**

8. Give an example of a ring having identity but a subring of this having a different identity. **(10 marks)**

9. If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R′, Prove that $\phi(1)$ is the unit element of R′ **(15 marks)**

10. Do the following sets form integral domains with respect to ordinary addition and multiplication? Is so, state if they are fields:

    (i) The set of numbers of the form $b\sqrt{2}$ with b rational

    (ii) The set of even integers.

    (iii) The set of positive integers. **(5+6+4=15 marks)**

2014

11. Let G be the set of all real $2\times 2\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$ where $xz \neq 0$ matrices where. Show that G is group under matrix multiplication. Let N denote the subset $\left\{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R\right\}$. Is N a normal subgroup of G? Justify your answer. **(10 marks)**

12. Show that Z_7 is a field. Then find $([5]+[6])^{-1}$ and $(-[4])^{-1}$ in Z_7 **(15 marks)**

13. Show that the set $\{a + b\omega : \omega^3 = 1\}$, where a and b are real numbers, is a field with respect to usual addition and multiplication. **(15 marks)**

14. Prove that the set $Q(\sqrt{5}) = \{a + b\sqrt{5} : a, b \in Q\}$ is commutative ring with identity. **(15 marks)**

2013

15. Show that the set of matrices $S = \left\{\begin{pmatrix} a & -b \\ b & a \end{pmatrix} a, b \in R\right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f: C \to S$ defined by $f(a+ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomporphism. (Here R is the set of real numbers and C is the set of comples numbers)? **(10 marks)**

16. Give an example of an infinite group in which every element has finite order **(10 marks)**

17. What are the orders of the following permutation in S_{10} ?$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$ and $(1\ \ 2\ \ 3\ \ 4\ \ 5)(6\ \ 7)$ **(10 marks)**

18. What is the maximal possible order of an element in S_{10}? Why ? Give an example of such an element. How many elements will there be in S_{10} of that order? **(13 marks)**

19. Let $J = \{a + ib / a, b \in Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J: Euclidean domain, principal ideal domain, and unique factorization domain ? Justify your answer **(15 marks)**

20. Let R^C= ring of all real value continuous functiosn on [0,1], under the operations $(f+g)x = f(x) + g(x), (fg)x = f(x)g(x)$. Let $M = \left\{f \in R^C / f\left(\frac{1}{2}\right) = 0\right\}$. Is M a maximal ideal of R? Justify your answer. **(15 marks)**

2012

21. How many elements of order 2 are there in the group of order 16 generated by a and b such that the order of a is 8, the order of b is 2 and $bab^{-1}=a^{-1}$. **(12 marks)**

22. How many conjugacy classes does the permutation groups S_5 of permutations 5 numbers have? Write down one element in each class (preferably in terms of cycles). **(15 marks)**

23. Is the ideal generated by 2 and X in the polynomial ring $Z[X]$ of polynomials in a single variable X with coefficients in the ring of integers Z, a principal ideal? Justify your answer **(15 marks)**

24. Describe the maximal ideals in the ring of Gaussian integers $z[i]=\{a+ib/a,b\in Z\}$. **(20 marks)**

2011

25. Show that the set $G=\{f_1,f_2,f_3,f_4,f_5,f_6\}$ of six transformations on the set of Complex numbers defined by

$$f_1(z)=z,f_2(z)=1-z,f_3(z)=\frac{z}{(1-z)},f_4(z)=\frac{1}{z},f_5(z)=\frac{1}{(1-z)},f_6(z)=\frac{(z-1)}{z}$$

is a nonabelian group of order 6 w.r.t. composition of mappings **(12 marks)**

26. Prove that a group of Prime order is abelian. **(6 marks)**

27. How many generators are there of the cylic group(G,.) of order 8? **(6 marks)**

28. Give an example of a group G in which every proper subgroup is cyclic but the group itself is not cyclic **(15 marks)**

29. Let F be the set of all real valued continuous functions defined on the closed interval [0,1]. Prove that (F, +, .) is a Commutative Ring with unity with respect to addition and multiplication of functions defined point wise as below:

$$\left.\begin{array}{l}(f+g)x=f(x)+g(x)\\ \text{and}\quad (fg)x=f(x)g(x)\end{array}\right\}x\in[0,1] \text{ where } f,g\in F$$ **(15 marks)**

30. Let a and b be elements of a group, with $a^2=e$, $b^6=e$ and $ab=b^4a$. Find the order of ab, and express its inverse in each of the forms a^mb^n and b^ma^n **(20 marks)**

2010

31. Let $G=R-\{-1\}$ be the set of all real numbers omitting –1. Define the binary relation * on G by $a*b=a+b+ab$. Show (G, *) is a group and it is abelian **(12 marks)**

32. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify. **(12 marks)**

33. Let (R^*, .) be the multiplicative group of non-zero reals and ($GL(n,R),X$) be the multiplicative group of n×n non-singular real matrices. Show that the quotient group $\frac{GL(n,R)}{SL(n,R)}$ and (R^*, .) are isomorphic where $SL(n,R)=\{A\in GL(n,R)/\det A=1\}$ What is the center of $GL(n,R)$ **(15 marks)**

34. Let $C = \{f : I = [0,1] \to R / f \text{ is continuous.}\}$ Show C is a commutative ring with 1 under point wise addition and multiplication. Determine whether C is an integral domain. Explain. **(15 marks)**

35. Consider the polynomial ring Q $[x]$. Show $p(x) = x^3 - 2$ is irreducible over Q. Let I be the ideal Q $[x]$ in generated by $p(x)$. Then show that $\frac{Q[x]}{I}$ is field and that each element of it is of the form $a_0 + a_1 t + a_2 t^2$ with a_0, a_1, a_2 in Q and t=x+1 **(15 marks)**

36. Show that the quotient ring $\frac{Z[i]}{1+3i}$ is isomorphic to the ring $\frac{Z}{10Z}$ where $Z[i]$ denotes the ring of Gaussian integers **(15 marks)**

2009

37. If R is the set of real numbers and R_+ *is* the set of positive real numbers, show that R under addition (R,+) and R_+ under multiplication (R_+,.) are isomorphic. Similarly if Q is set of rational numbers and Q+ is the set of positive rational numbers, are (Q,+) and (Q+,.) isomorphic? Justify your answer. **(4+8=12 marks)**

38. Determine the number of homomorphisms from the addititve group Z_{15} to the additive group Z_{10} (Z_n is the cyclic group of order n) **(12 marks)**

39. How many proper, non-zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $Z_{12} \oplus Z_{12}$ have? Why? **(2+3+4+6=15 marks)**

40. Show that the alternating group of four letters A_4 has no subgroup of order 6. **(15 marks)**

41. Show that $Z[X]$ is a unique factorization domain that is not a principal ideal domain (Z is the ring of integers). Is it possible to give an example of principal ideal domain that is not a unique factorization domain? ($Z[X]$) is the ring of polynomial in the variable X with integer.) **(15 marks)**

42. How many elements does the quotient ring $\frac{Z_5[X]}{X^2+1}$ have? Is it an integral domain? Justify your answers. **(15 marks)**

2008

43. Let R_0 be the set of all real numbers except zero. Define a binary operation * on R_0 as $a * b = |a| b\ |a|$ where denotes absolute value of a. Does (R_0, *) form a group? Examine. **(12 marks)**

44. Suppose that there is a positive even integer n such that $a^n = a$ for all the elements a of some ring R. Show that a+a=0 for all $a \in R$ and $a + b = 0 \Rightarrow a = b$ for all $a, b \in R$ **(12 marks)**

45. Let G and $\overline{G}$ be two groups and let $\phi : G \to \overline{G}$ be a homomorphism. For any element $a \in G$

(i) Prove that $O(\phi(a)) / O(a)$

(ii) Ker ϕ is normal subgroup of G. **(15 marks)**

46. Let R be a ring with unity. If the product of any two non-zero elements is non-zero. Then prove that $ab = 1 \Rightarrow ba = 1$. Whether Z_6 has the above property or not explain. Is Z_6 an integral domain? **(15 marks)**

47. Prove that every integral domain can be embedded in a field. **(15 marks)**

48. Show that any maximal ideal in the commutative ring $F[x]$ of polynomial over a field F is the principal ideal generated by an irreducible polynomail. **(15 marks)**

2 007

49. If in a group G, $a^5 = e$, e is the identify element of G $aba^{-1} = b^2$ for $a, b \in G$, then find the order of b **(12 marks)**

50. Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in Z$. Show that R is a ring under matrix addition and multiplication $\left\{ A = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}, a, b \in Z \right\}$. Then show that A is a left ideal of R but not a right ideal of R. **(12 marks)**

51. (i) Prove that there exists no simple group or order 48.

(ii) $1 + \sqrt{-3}$ and $Z\left[\sqrt{-3}\right]$ is an irreducible element, but not prime. Justify your answer. **(15 marks)**

52. Show that in the ring $R = \left\{ a + b\sqrt{-5} / a, b \in Z \right\}$. The element $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R, where $\gamma = 7\left(1 + 2\sqrt{-5}\right)$ **(30 marks)**

2006

53. Let S be the set of all real numbers except –1. Define on S by $a * b = a + b + ab$. Is $(S,*)$ a group? Find the solution of the equation $2 * x * 3 = 7$ in S. **(12 marks)**

54. If G is a group of real numbers under addition and N is the subgroup of G consisting of integers, Prove that $\frac{G}{N}$ is isomorphic to the group H of all complex numbers of absolute value 1 under multiplication **(12 marks)**

55. (i) Let $O(G)$= 108. Show that there exists a normal subgroup or order 27 or 9.

(ii) Let G be the set of all those ordered paris (a,b) real numbers for which $a \neq 0$ and define in G, an operation as follows: $(a,b) \otimes (c,d) = (ac, bc + d)$ Examine whether G is a group w.r.t the operation $\otimes$ If it is a group, is G abelian? **(10 marks)**

56. Show that $Z\left[\sqrt{2}\right] = a\left\{ a + b\sqrt{2} : a, b \in Z \right\}$ is a Euclidean domain. **(30 marks)**

2005

57. If M and N are normal subgroups of a group G such that $M \cap N = [e]$, show that every element of M commutes with every element of N. **(12 marks)**

58. Show that $(1+i)$ is a prime element in the ring R of Gaussian integers. **(12 marks)**

59. Let H and K be two subgroups of a finite group G such that $|H| > \left|\sqrt{|G|}\right|$ and $|K| > \left|\sqrt{|G|}\right|$. Prove that $H \cap K \neq \{e\}$. **(15 marks)**

60. If $f : G \to G'$ is an isomorphism, prove that the order $a \in G$ of is equal to the order of $f(a)$ **(15 marks)**

61. Prove that any polynomial ring $F[x]$ over a field F is U.F.D **(30 marks)**

2004

62. If p is prime number of the form $4n+1$, n being a natural number, then show that con gruence $x^2 \equiv -1\ (mod\ p)$ is solvable. **(12 marks)**

63. Let G be group such that of all $a, b \in G$ (i) ab = ba (ii) $\left(O(a), O(b)\right) = 1$ then show that $O(ab) = O(a)\ O(b)$ **(12 marks)**

64. Verify that the set E of the four roots of $x^4 - 1 = 0$ forms a multiplicative group. Also prove that a transformation T, $T(n) = i^n$ is a homomorphism from I_+ (Group of all inte gers with addition) onto E under multiplication. **(10 marks)**

65. Prove that if cancellation law holds for a ring R then $a(\neq 0) \in R$ is not a zero divisor and conversely **(10 marks)**

66. The residue class ring $\frac{Z}{(m)}$ is a field iff m is a prime integer. **(15 marks)**

67. Define irreducible element and prime element in an integral domain D with units. Prove that every prime element in D is irreducible and converse of this is not (in general) true. **(25 marks)**

2003

68. If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G **(12 marks)**

69. Show that the ring $Z[i] = \{a + ib / a, b \in Z, i = \sqrt{-1}\}$ of Gaussian integers is a Euclidean domain **(12 marks)**

70. Let R be the ring of all real-valued continuous functions on the closed interval [0,1]. Let $M = \left\{ f(x) \in R / f\left(\frac{1}{3}\right) = 0 \right\}$. Show that M is a maximal ideal of R **(10 marks)**

71. Let M and N be two ideals of a ring R. Show that $M \cup N$ is an ideal of R if and only if either $M \subseteq N$ or $N \subseteq M$ **(10 marks)**

72. Show that $Q(\sqrt{3}, i)$ is a spliting field for $x^5 - 3x^3 + x^2 - 3$ where Q is the field of rational numbers **(15 marks)**

73. Prove that x^2+x+4 is irreducible over F the field to integers modulo 11 and prove further that $\frac{F[x]}{(x^2+x+4)}$ is a field having 121 elements. **(15 marks)**

74. Let R be a unique factoriztion domain (U.F.D), then prove that $R[x]$ is also U.F.D **(10 marks)**

2002

75. Show that a group of order 35 is cyclic. **(12 marks)**

76. Show that polynomial $25x^4+9x^3+3x+3$ is irreducible over the field of rational numbers **(12 marks)**

77. Show that a group of p^2 is abelian, where p is a prime number. **(10 marks)**

78. Prove that a group of order 42 has a normal subgroup of order 7. **(10 marks)**

79. Prove that in the ring $F[x]$ of polynomial over a field F, the ideal $I = |p(x)|$ is maximal if and only if the polynomail p(x) is irreducible over F. **(20 marks)**

80. Show that every finite integral domain is a field **(10 marks)**

81. Let F be a field with q elements. Let E be a finite extension of degree n over F. Show that E has q^n elements. **(10 marks)**

2001

82. Let K be a field and G be a finite subgroup of the multiplicative group of non-zero elements of K. Show that G is a cyclic group. **(12 marks)**

83. Prove that the polynomial $1+x+x^2+x^3+ \dots +x^{p-1}$ where p is prime number is irreducible over the field of rational numbers. **(12 marks)**

84. Let N be a normal subgroup of a group G. Show that $\frac{G}{N}$ is abelain if and only if for all $x, y \in G$, $xyz^{-1} \in N$ **(20 marks)**

85. If R is a commutative ring with unit element and M is an ideal of R, then show that maximal ideal of R if and only if $\frac{R}{M}$ is a field. **(20 marks)**

86. Prove that every finite extension of a field is an algebraic externsion. Give an example to show that the converse is not true. **(20 marks)**

2000

87. Let n be a fixed positive integer and Let Z_n be the ring of intergers modulo n. Let $G = \{a \in Z_n a | a \neq 0\}$ and a is relatively prime to n. Show that G is a group under multipli cation defined in Z_n. Hence, or otherwise, show that $a^{\phi(n)} = a(mod n)$ for all integers a relatively prime to n where $\phi(n)$ denotes the number of positive integers that are less than n and are relatively prime to n **(20 marks)**

88. Let M be a subgroup and N a normal subgroup of G. Show that MN is a subgroup G and $\frac{MN}{N}$ is isomorphic to $\frac{M}{M \cap N}$ **(20 marks)**

89. Let F be a finite field. Show that the characteristic of F must be a prime integer p and the number of elements in F must be p^m for some positive integer m. **(20 marks)**

90. Let F be a field and $F[x]$ denote the set of all polynomials defined over F. If $f(x)$ is an irreducible polynomial in $F[x]$, show that the ideal generated by $f(x)$ in $F[x]$ is maximal and $\frac{F[x]}{f(x)}$ is a field. **(20 marks)**

91. Show that any finite commutative ring with no zero divisors must be a field. **(20 marks)**

1999

92. If ϕ is a homomorphism of G into $\overline{G}$ with kernel K, then show that K is a normal sub group of G. **(20 marks)**

93. If p is prime number and $p^{\alpha} / O(G)$, , then prove that G has a subgroup of order p^{α}. **(20 marks)**

94. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field. **(20 marks)**

1998

95. Prove that if a group has only four elements then it must be abelian. **(20 marks)**

96. If H and K are subgroups of a group G then show that HK is a subgroup of G if and only if only $HK=KH$. **(20 marks)**

97. Let $(R,+,.)$ be a system satisfying all the axioms for a ring with unity with the possible exception of $a+b=b+a$. Prove that $(R,+,.)$ is a ring. **(20 marks)**

98. If p is prime then prove that Z_p is a field. Discuss the case when p is not a prime number. **(20 marks)**

99. Let D be a principal domain. Show that every element that its neither zero nor a unit in D is a product of irreducible. **(20 marks)**

1997

100. Show that a necessary and sufficient condition for a subset H of a group G to be a subgroup is $HH^{-1}=H$. **(20 marks)**

101. Show that the order of each subgroup of a finite group is a divisor of the order of the group. **(20 marks)**

102. In a group G, the commutator $(a,b), a, b \in G$ is the element $aba^{-1}b^{-1}$ and the smallest subgroup containing all commutators is called the commutator subgroup of G. Show that a quotient group $\frac{G}{H}$ is abelian if and only if H contains the commutator subgroup of G. **(20 marks)**

103. If $x^2=x$ for all x in a ring R, show that R is commutative. Give an example to show that the converse is not true. **(20 marks)**

104. Show that an ideal S of the ring of integers Z is maximal ideal if and only if S is generated by a prime integer. **(20 marks)**

105. Show that in an integral domain every prime element is irreducible. Give an example to show that the converse is not true. **(20 marks)**

1996

106. Let R be the set of real numbers and $G=\{(a,b) a,b \in R, a \neq 0\}$. $G \times G \to G$ is defined by $(a,b)*(c,d)=(ac,bc+d)$. Show that (G,*) is a group. Is it abelian? Is (H,*) a Subgroup of (G,*) when H={(1,b)/b∈R}? **(20 marks)**

107. Let f be a homomorphism of a group G onto a group G' with kernel H. For each sub group K' of G define K by k={x∈G/f(x)∈K′}. Prove that

(i) K' is isomorphic to $\frac{K}{H}$

(ii) $\frac{G}{K}$ is isomorphic to $\frac{G'}{K'}$ **(20 marks)**

108. Prove that a normal subgroup H of a group G is maximal, if and only if the quotient group $\frac{G}{H}$ is simple. **(20 marks)**

109. In a ring R, Prove that cancellation laws hold. If and only if R has no zero divisors. **(20 marks)**

110. If S is an ideal of a ring R and T any subring of R, then prove that S is an ideal of $S+T=\{s+t | s \in S, t \in T\}$. **(20 marks)**

111. Prove that the polynomial x^2+x+4 is irreducible over the field of integers modulo 11. **(20 marks)**

1995

112. Let G be a finite set closed under an associative binary operation such that $ab=ac \Rightarrow b=c$ and $ba=ca \Rightarrow b=c$ for all $a,b,c \in G$ prove that G is a group. **(20 marks)**

113. Let G be group of order p^n. Where p is a prime number and $n>0$. Let H be a proper subgroup of G and $N(H)=\{x \in G : x^{-1}hx \in H \ \forall h \in H\}$. Prove that $N(H) \neq H$ **(20 marks)**

114. Show that a group of order 112 is not simple **(20 marks)**

115. Let R be a ring with identity. Suppose there is an element a of R which has more than one right inverse. Prove that a has infinitely many right inverses. **(20 marks)**

116. Let F be a field and let $p(x)$ be an irreducible polynomial over F. Let $\langle p(x) \rangle$ be the ideal generated by p(x). Prove that $\langle p(x) \rangle$ is a maximal ideal. **(20 marks)**

117. Let F be a field of characteristic $p \neq 0$. Let $F(x)$ be the polynomial ring. Suppose $f(x) = a_0 + a_1x + a_2x^2 + ...a_nx^n$ is an element of $F(x)$. Define $f(x) = a_1 + 2a_2x + 3a_3x^2 + ... + na_nx^{n-1}$. If f(x)=0, prove that there exist $g(x) \in F(x)$ such that $f(x) = g(x^p)$. **(20 marks)**

1994

118. If G is a group such that $(ab)^n = a^nb^n$ for three consecutive integers n for all $a,b \in G$, then prove that G is abelian. **(20 marks)**

119. Can a group of order 42 be simple? Justify your claim **(20 marks)**

120. Show that the additive group of integers modulo 4. Is isomorphic to the multiplicative group of the non-zero elements of integers modulo 5. State the two isomorphisms

121. Find all the units of the integral domain of Gaussian integers. **(20 marks)**

122. Prove or disprove the statment: The polynomial ring $I[x]$ over the ring of integers is a principal ideal ring. **(20 marks)**

123. If R is an integral domain (not necessarily a unique factorization domain) and F is its field of quotients, then show that any element $f(x)$ in $F(x)$ is of the form $f(x) = \frac{f_0(x)}{a}$ where $f_0(x) \in R[x], a \in R$. **(20 marks)**

1993

124. If G is a cyclic group of order n and p dividers n, then prove that there is a homomor phism of G onto a cyclic group of order p. What is the Kernel of hommorphism? **(20 marks)**

125. Show that a group of order 56 cannot be simple **(20 marks)**

126. Suppose that H,K are normal subgroups of a finite group G with H a normal subgroup of K. If $P = \frac{K}{H}, S = \frac{G}{H}$, then Prove that the quotient groups $\frac{S}{P}$ and $\frac{G}{K}$ are isomorphic. **(20 marks)**

127. If Z is the set of integers then show that $Z\left[\sqrt{-3}\right] = \left\{a + \sqrt{-3}\,b : a,b \in Z\right\}$ is not a unique factorization domain **(20 marks)**

128. Construct the addition and multiplication table for $\frac{Z_3[x]}{\langle x^2+1\rangle}$ where Z_3 is the set of inte gers modulo 3 and $\langle x^2+1\rangle$ is the ideal generated by (x^2+1) in $Z_3[x]$. **(20 marks)**

129. Let Q be the set of rational number and Q($21^{1/2}, 2^{1/3}$) the smallest extension field of Q containing $2^{1/2}$, $2^{1/3}$. Find the basis for Q($2^{1/2}, 2^{1/3}$) over Q. **(20 marks)**

1992

130. If H is a cyclic normal subgroup of a group G, then show that every subgroup of H is normal in G. **(20 marks)**

131. Show that no group of order 30 is simple **(20 marks)**

132. If p is the smallest prime factor of the order of a finite group G, prove that any subgroup of index p is normal. **(20 marks)**

133. If R is unique factorization domain, then prove that any $f \in R[x]$ is an irreducible ele ment of $R[x]$, if and only if either f is an irreducible element of R or f is an irreducible polynomial in $R[x]$. **(20 marks)**

134. Prove that x^2+1 and x^2+x+4 are irreducible over F, the field of integer modulo 11. Prove also that $\frac{F[x]}{\langle x^2+1\rangle}$ and $\frac{F[x]}{\langle x^2+x+4\rangle}$ are isomorphic fields each having 121 elements.

(20 marks)

135. Find the degree of splitting field $x^5-3x^3+x^2-3$ over Q, the field of rational numbers. **(20 marks)**