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MATHEMATICS

REAL ANALYSIS

Previous year Questions from **1992 To 2017**

Syllabus

Real number system as an ordered field with least upper bound property;Sequences, limit of a sequence, Cauchy sequence, completeness of real line; Series and its convergence, absolute and conditional convergence of series of real and complex terms, rearrangement of series. Continuity and uniform continuity of functions, properties of continuous functions on compact sets. Riemann integral, improper integrals; Fundamental theorems of integral calculus.Uniform convergence, continuity, differentiability and integrability for sequences and series of functions; Partial derivatives of functions of several (two or three) variables, maxima and minima.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

2017

1. Let x_1=2 and $x_{n+1}=\sqrt{x_n+20}, n=1,2,3....$ Show that the sequence $x_1,x_2,x_3...$ is convergent. **[10 marks]**

2. Find the supermum and the infimum of $\frac{x}{\sin x}$ on the interval $\left(0,\frac{\pi}{2}\right]$. **[10 marks]**

3. Determine all entire functions f(z) such that 0 is a removable singularity of $f\left(\frac{1}{z}\right)$ **[10 marks]**

4. Let $f(t)=\int_0^t [x]dx,$
where $[x]$ denotes the largest integer less than or equal to x.
(i) Determine all the real numbers t at which f is differentiable.
(ii) Determine all the real numbers t at which f is continuous but not differentiable. **[15 marks]**

5. Let $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series of real numbers. Show that there is a rearrangement $\sum_{n=1}^{\infty} x_{\pi(n)}$ of the series $\sum_{n=1}^{\infty} x_n$ that converges to 100. **[20 marks]**

2016

6. For the function f: $(0,\infty)\to R$ given $f(x)=x^2\sin\frac{1}{x}, 0<x<\infty$ Show that there is a differentiable function $g:R\to R$ that extends f **[10 marks]**

7. Two sequences $\{x_n\}$ and $\{y_n\}$ are defined inductively by the following:
$x_1=\frac{1}{2}, y_1=1, x_n=\sqrt{x_{n-1}y_{n-1}}, n=2,3,4,...\frac{1}{y_n}=\frac{1}{2}\left(\frac{1}{x_n}+\frac{1}{y_{n-1}}\right), n=2,3,4,...$ and Prove that $x_{n-1}<x_n<y_n<y_{n-1}, n=2,3,4,.....$ and deduce that both the sequences convergence to the same limit l where $\frac{1}{2}<l<1$. **[10 marks]**

8. Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$ conditionally convergent (if you use any theorem (s) to show it then you must give a proof of that theorem(s). **[15 marks]**

9. Find the relative maximum minimum values of the function $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$ **[15 marks]**

10. Let $f:R\to R$ be a continuous function such $\lim_{x\to+\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$ exist and are finite. Prove that f is uniformly continuous on IR **[15 marks]**

2015

11. Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{n^2+1}\right)$
10 marks]

12. Is the function $f(x)=\begin{cases}\frac{1}{n}, & \frac{1}{n+1}<x\le\frac{1}{n}\\ 0 & x=0\end{cases}$ Rienmann Integrable? If yes, obtain the value of $\int_0^1 f(x)\,dx$ **[15 marks]**

13. Test the series of functions $\sum_{n=1}^{\infty}\frac{nx}{1+n^2x^2}$ for uniform convergence **[15 marks]**

14. Find the absolute maximum and minimum values of the function $f(x,y)=x^2+3y^2-y$ over the region $x^2+2y^2\le 1$ **[15 marks]**

2014

15. Test the convergence of the improper integral $\int_1^{\infty}\frac{dx}{x^2\left(1+e^{-x}\right)}$ **[10 marks]**

16. Integrate $\int_0^1 f(x)\,dx$, where $f(x)=\begin{cases}2x\sin\frac{1}{x}-\cos\frac{1}{x}, & x\in[0,1]\\ 0 & x=0\end{cases}$ **[15 marks]**

17. Obtain $\frac{\partial^2 f(0,0)}{\partial x\partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y\partial x}$ for the function

$f(x,y)=\begin{cases}\frac{xy\left(3x^2-2y^2\right)}{x^2+y^2}, & (x,y)\ne(0,0)\\ 0 & ,(x,y)\ne(0,0)\end{cases}$ Also, discuss the continuity $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ of at (0,0) **[15 marks]**

18. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$ by the method of Lagrange multipliers. **[15 marks]**

2013

19. Let $f(x)=\begin{cases}\frac{x^2}{2}+4 & if\ \ x\ge 0\\ \frac{-x^2}{2}+2 & if\ \ x<0\end{cases}$ is f Riemann integrable in the interval [–1,2]? Why ?

Does there exist a function g such that $g'(x)=f(x)$? Justify your answer. **[10 marks]**

20. Show that the series $\sum_{1}^{\infty}\frac{(-1)^{n-1}}{n+x^2}$, is uniformly convergent but not absolutely for all real values of x. **[13 marks]**

21. Show that every open subset of R is countable union of disjoint open intervals **[14 marks]**

22. Let $[x]$ denote the integer part of the real number x, i.e., if n $\leq x <$ n+1 where n is an integer, then $[x]=n$. Is the function f(x) = $[x]^2$+3 Rienman integrable in the function in [–1,2]? if not, explain why. If it is integrable, compute $\int_{-1}^{2}\left([x]^2+3\right)dx$ **[10 marks]**

2012

23. Let, $f_n(x)=\begin{cases}0, & if\ x<\dfrac{1}{n+1}\\ \sin\dfrac{\pi}{x}, & if\ x<\dfrac{1}{n+1}\leq x\leq\dfrac{1}{n},\\ 0, & if\ x>\dfrac{1}{n}.\end{cases}$ Show that $f_n(x)$ converges to a continuous function but not uniformly. **[12 marks]**

24. Show that the series $\sum_{n=1}^{\infty}\left(\frac{\pi}{\pi+1}\right)^n n^6$ is convergent **[12 marks]**

25. Let $f(x,y)=\begin{cases}\dfrac{(x+y)^2}{x^2+y^2}, & if\ f(x,y)\neq(0,0)\\ 1, & if\ (x,y)=(0,0)\end{cases}$ $Show\ that\ \dfrac{\partial f}{\partial x}\ and\ \dfrac{\partial f}{\partial y}$ exist at (0,0) though *f(x,y)* is not continuous at (0,0). **[15 marks]**

26. Find the minimum distance of the line given by the planes $3x+4y+5z$=7 and $x-z$=9 and from the origin, by the method of Lagrange's multipliers. **[15 marks]**

27. Let *f(x)* be differentiable on [0,1], such that f(1)=f(0)=0 and $\int_0^1 f^2(x)\,dx=1$. Prove that $\int_0^1 x\,f(x)\,f'(x)\,dx=-\frac{1}{2}$ **[15 marks]**

28. Give an example of a function $f(x)$, that is not Riemann integrable but $|f(x)|$ is Riemann integrable. Justify your answer. **[20 marks]**

2011

29. Let S=(0,1) and f be defined by $f(x)=\frac{1}{x}$ where $0 \le x \le 1$ (in R). Is f uniformly continuous on S? justify your answer. **[12 marks]**

30. Let $f_n(x)=nx(1-x)^n$, $x \in [0,1]$. Examine the uniform convergence of $\{f_n(x)\}$ on [0,1] **[15 marks]**

31. Find the shortest distance from the origin (0,0) to the hyperbola $x^2+8xy+7y^2=225$ **[15 marks]**

32. Show that the series for which the sum of first n terms $f_n(x)=\frac{nx}{1+n^2x^2}, 0 \le x \le 1$ cannot be differentiated term-by-term at x=0. What happens at $x \neq 0$? **[15 marks]**

33. Show that if $S(x)=\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$, then its derivative $S'(x)=-2x\sum_{n=1}^{\infty}\frac{1}{n^2\left(1+nx^2\right)^2}$, for all x **[20 marks]**

2010

34. Discuss the convergence of the sequence $\{x_n\}$ where $X_n=\frac{\sin\left(\frac{n\pi}{2}\right)}{8}$ **[12 marks]**

35. Define $\{x_n\}$ by x_1=5 and $x_{n+1}=\sqrt{4+x_n}$ for n>1 Show that the sequence converges to $\left(\frac{1+\sqrt{17}}{2}\right)$ **[12 marks]**

36. Define the function $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if\ x \neq 0 \\ 0 & if\ x=0 \end{cases}$. Find f′(x). Is f′(x) continuous at x=0? Justify your answer. **[15 marks]**

37. Consider the series $\sum_{n=0}^{\infty}\frac{x^2}{\left(1+x^2\right)^2}$. Find the values of x for which it is convergent and also the sum function. Is the converse uniform? Justify your answer. **[15 marks]**

38. Let $f_n(x)=x^n$ on $-1 < x \leq 1$ for n=1,2.... Find the limit function. Is the convergence uni form? Justify your answer. **[15 marks]**

39. State Roll's theorem. Use it to prove that between two roots of $e^x\cos x=1$ there will be a root $e^x \sin x=1$ **[2+10=12 marks]**

40. Let $f(x)=\begin{cases} \frac{|x|}{2}+1 & if\ x<1 \\ \frac{x}{2}+1 & if\ 1\leq x<2 \\ -\frac{|x|}{2}+1 & if\ 2\leq x \end{cases}$ What are the points of discontinuity of f, if any? What are the points where f is not differentiable, if any? Justify yours answer. **[12 marks]**

41. Show that the series $\left(\frac{1}{3}\right)^2+\left(\frac{1.4}{3.6}\right)^2+....+\left(\frac{1.4.7.....(3n-2)}{3.6.9......3n}\right)^2+.....$ converges **[15 marks]**

42. Show that if f:$[a,b]\to R$ is a continuous function then $f([a,b])=[c,d]$ form some real numbers c and d, $c\leq d$. **[15 marks]**

43. Show that : $\lim\limits_{x\to 1}\sum\limits_{n=1}^{\infty}\frac{n^2x^2}{n^4+x^4}=\sum\limits_{n=1}^{\infty}\frac{n^2}{n^4+1}$ Justify all steps of your answer by quoting the theorems you are using **[15 marks]**

44. Show that a bounded infinite subset R must have a limit point **[15 marks]**

2008

45. (i) For x>0, show $\frac{x}{1+x}<\log(1+x)<x$ **[6 marks]**

(ii) Let $T=\left\{\frac{1}{n},n\in N\right\}\cup\left\{1+\frac{3}{2n},n\in N\right\}\cup\left\{6-\frac{1}{3n},n\in N\right\}$. Find derived set T of T.

Also find Supremum of T and greatest number of T. **[6 marks]**

46. If f:$R\to R$ is continuous and $f(x+y)=f(x)+f(y)$, for all $x,y\in R$ then show that $f(x)=xf(1)$ for all $x\in R$. **[12 marks]**

47. Discuss the convergence of the series $\frac{x}{2}+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+....,x>0$ **[15 marks]**

48. Show that the series $\sum\frac{1}{n(n+1)}$ is equivalent to $\frac{1}{2}\prod\limits_{2}^{\infty}\left(1+\frac{1}{n^2-1}\right)$ **[15 marks]**

49. Let f be a continuous function on [0,1]. Using first Mean Value theorem on Integration, prove that $\lim\limits_{n\to\infty}\int_0^1\frac{nf(x)}{1+n^2x^2}dx=\frac{\pi}{2}f(0)$ **[15 marks]**

50. (i) Prove that the sets A=[0,1], B=(0,1) are equivalent sets. **[6 marks]**

(ii) Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}, x \in \left(0, \frac{\pi}{2}\right)$ **[9 marks]**

2007

51. Show that the function given by $f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is not continuous at (0,0) but its partial derivatives f_x and f_y exists at (0,0) **[12 marks]**

52. Using Lagrange's mean value theorem, show that $|\cos b - \cos a| \leq |b-a|$ **[12 marks]**

53. Given a positive real number α and any natural number n, prove that there exists one and only positive real number ξ such that $\xi^2 = a$ **[20 marks]**

54. Find the volume of the solid in the first octant bounded by the paraboloid $z=36-4x^2-9y^2$ **[20 marks]**

55. Rearrange the series $\sum(-1)^{n+1}\frac{1}{n}$ converge to 1 **[20 marks]**

2006

56. Examine the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$ **[12 marks]**

57. Prove that the function f defined by $f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational} \end{cases}$ is nowhere continuous. **[12 marks]**

58. A twice differentiable function f is such that $f(a)=f(b)=0$ and $f(c)>0$ for $a<c<b$. Prove that there is at least one value ξ, a < ζ< b for which $f''(\zeta)<0$. **[20 marks]**

59. Show that the function given by $f(x,y) = \begin{cases} \frac{x^2+2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

(i) is continuous at(0,0)
(ii) possesses partial derivative $f_x(0,0)$ and $f_y(0,0)$ **[20 marks]**

60. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **[20 marks]**

2005

61. If u,v,w are the roots of the equation in λ and $\frac{x}{a+\lambda}+\frac{y}{b+\lambda}+\frac{z}{c+\lambda}=1$,

evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ **[12 marks]**

62. Evaluate $\iiint \ln(x+y+z)\,dxdydz$ The integral being extended over all positive values of x,y,z such that $x+y+z \le 1$ **[12 marks]**

63. If f' and g' exist for every $x \in [a,b]$ and if $g'(x)$ does not vanish anywhere (a,b), show that there exists c in (a,b) such that $\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f'(c)}{g'(c)}$ **[30 marks]**

64. Show that $\int_0^\infty e^{-t}t^{n-1}dt$ is an improper integral which converges for $n > 0$ **[30 marks]**

2004

65. Show that the function f(x) defined as : $f(x)=\frac{1}{2^n}, \frac{1}{2^{n+1}} \le x \le \frac{1}{2^n}$, n=0,1,2 .... and $f(0)=0$ is integrable in [0,1], although it has an infinite number of points of discontinuity. Show that $\int_0^1 f(x)\,dx = \frac{2}{3}$ **[12 marks]**

66. Show that the function $f(x)$ defined on by: $f(x)=\begin{cases} x & \text{when } x \text{ is irrational} \\ -x & \text{when } x \text{ is rational}\end{cases}$ is continuous only at $x=0$ **[12 marks]**

67. If (x,y,z) be the lengths of perpendiculars drawn from any interior point P of triangle ABC on the sides BC, CA and AB respectively, then find the minimum value of $x^2+y^2+z^2$, the sides of the triangle ABC being a,b,c. **[20 marks]**

68. Find the volume bounded by the paraboloid $x^2+y^2=az$, the cylinder $x^2+y^2=2ay$ and the plane $z=0$ **[20 marks]**

69. Let $f(x) \ge g(x)$ for every x in $[a,b]$ and f and g are both bounded and Riemann inte grable on $[a,b]$. At a point $c\in[a,b]$, let f and g be continuous and $f(c) > g(c)$ then prove that $\int_a^b f(x)\,dx > \int_a^b g(x)dx$ and hence show that $-\frac{1}{2} < \int_a^b \frac{x^3 \cos 5x}{2+x^2}dx < \frac{1}{2}$ **[20 marks]**

2003

70. Let a be a positive real number and $\{x_n\}$ sequence of rational numbers such that $\lim_{n\to\infty} ax_n = 1$ **[12 marks]**

71. If a continuous function of x satisfies the functional equation $f(x+y) = f(x) + f(y)$ then show that $f(x) = ax$ where a is a constant. **[12 marks]**

72. Show that the maximum value of $x^2y^2z^2$ subject to condition $x^2+y^2+z^2=c^2$ is $\frac{c^2}{27}$. Interpret the result **[20 marks]**

73. The axes of two equal cylinders intersect at right angles. If a be their radius, then find the volume common to the cylinder by the method of multiple integrals. **[20 marks]**

74. Show that $\int_0^{\infty} \frac{dx}{1+x^2 \sin^2 x}$ is divergent **[20 marks]**

2002

75. Prove that the integral $\int_0^{\infty} x^{m-1} e^{-x} dx$ is convergent if and only if $m>0$. **[12 marks]**

76. Find all the positive values of a for which the series $\sum_{n=1}^{\infty} \frac{(an)^n}{n!}$ converges. **[12 marks]**

77. Test uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^p}$, where $p>0$ **[20 marks]**

78. Obtain the maxima and minima of $x^2+y^2+z^2-yz-zx-xy$ subject to condition $x^2+y^2+z^2-2x+2y+6z+9=0$ **[25 marks]**

79. A solid hemisphere H of radius 'a' has density ρ depending on the distance R from the center and is given by $\rho = k(2a-R)$ where k is a constant. Find the mass of the hemisphere by the method of multiple integrals **[15 marks]**

2001

80. Show that $\int_0^{\pi/2} \frac{x^n}{\sin^m x} dx$ exists if and only if $m<n+1$ **[12 marks]**

81. If $\lim_{n\to\infty} a_n = l$, then prove that $\lim_{n\to\infty} \frac{a_1+a_2+\ldots a_n}{n} = l,$ **[12 marks]**

82. A function f is defined in the interval (a,b) as follows

$$f(x) = \begin{cases} \frac{1}{q^2} & \text{when } x = \frac{p}{q} \\ \frac{1}{q^3} & \text{when } x = \sqrt{\frac{p}{q}} \end{cases}$$

where p,q relatively prime integers. $f(x)=0$ for all other values of x. is f Riemann integrable? justify your answer. **[20 marks]**

83. Show that $U=xy+yz+zx$ has a maximum value when the three variables are connected by the relation $ax+by+cz=1$ and a,b,c are positive constants satisfying the condition $2(ab+bc+ca)>(a^2+b^2+c^2)$ **[25 marks]**

84. Evaluate $\iiint (ax^2+by^2+cz^2)\, dxdydz$ taken throughout the region $x^2+y^2+z^2 \le R^2$ **[15 marks]**

2000

85. Given that the terms of a sequence $\{a_n\}$ are such that each a_k, $k \le 3$, is the arithmetic mean of its two immediately preceding terms. Show that the sequence converges. Also find the limit of the sequence. **[12 marks]**

86. Determin the values of x for which infinite product $\prod_{n=0}^{\infty}\left(1+\frac{1}{x^{2n}}\right)$ converges absolutely. Find its value whenever it converges. **[12 marks]**

87. Suppose f is twice differentiable real valued function in $(0,\infty)$ and M_0, M_1 and M_2 the least upper bounds of $|f(x)|, |f'(x)| \text{ and } |f''(x)|$ respectively in $(0,\infty)$. Prove for each $x>0$, $h>0$ that $f'(x)=\frac{1}{2h}[f(x+2h)-f(x)]-hf'(u)$ for some $u\in(x,x+2h)$. Hence show that $M_1^2 \le 4M_0M_2$ **[20 marks]**

88. Evaluate $\iint_S \left(x^3dydz+x^2ydzdx+x^2zdxdy\right)$ by transforming into triple integral where S is the closed surface formed by the cylinder $x^2+y^2=a^2$, $0 \le z \le b$ and the circular disc $x^2+y^2 \le a^2$, z=0 and $x^2+y^2 \le a^2$, $z=b$ **[20 marks]**

1999

89. Let A be a subset of the metric space (M,ρ). If (A,ρ) is compact, then show that A is a closed subset of (M,ρ) **[20 marks]**

90. A sequence $\{S_n\}$ is defined by the recursion formula $S_{n+1}=\sqrt{3S_n}, S_1=1$. Does this sequence converge? if so, find lim S_n. **[20 marks]**

91. Test for convergence the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$ **[20 marks]**

92. Find the shortest distance from the origin to the hyperbola $x^2+8xy+7y^2=225, z=0$ **[20 marks]**

93. Show that the double integral $\iint_R \frac{x-y}{(x+y)^3}dxdy$ does not exist over R = [0,1;0,1] **[20 marks]**

94. Verify the Gauss divergence theorem for $\overline{F}=4x\hat{e}_x-2y^2\hat{e}_y+z^2\hat{e}_z$ taken over the region bounded by x^2+y^2=-4, z=0 and z=3 where $\hat{e}_x, \hat{e}_y, \hat{e}_z$ are unit vectors along x, y and z–directions respectively. **[20 marks]**

1998

95. Let X be a metric space and $E \subset X$. Show that **[20 marks]**
(i) Interior of E is the largest open set contained in E
(ii) Boundary of E = (closure of E) $\cap$ (Closure of $X–E$)

96. Let (X,d) and (Y,e) be metric spaces with X compact and f:$X\to Y$ continuous. Show that f is uniformly continuous. **[20 marks]**

97. Show that the function $f(x,y)=2x^4-3x^2y+y^2$ has (0,0) as the only critical point but the function neither has a minima nor maxima at (0,0) **[20 marks]**

98. Test the convergence of the integral $\int_0^\infty e^{-ax}\frac{\sin x}{x}dx,\ a\ge 0$ **[20 marks]**

99. Test the series $\sum_{n=1}^{\infty} \frac{x}{\left(n+x^2\right)^2}$ for uniform convergence. **[20 marks]**

100. Let $f(x) = x$ and $g(x) = x^2$. Does $\int_0^1 fog$ exist? If it exists then find its value

[20 marks]

1997

101. Show that a non-empty set P in R^n each of whose points is a limit -point is uncountable. **[20 marks]**

102. Show that $\iiint_D xyz\, dxdydx = \frac{a^2b^2c^2}{6}$ where domain D is given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \le 1$

[20 marks]

103. If $u = \sin^{-1}\left[\left(x^2+y^2\right)^{1/5}\right]$, Prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left(2\tan^2 u-3\right)$

[20 marks]

1996

104. Let F be the set of all real valued bounded continuous functions defined on the closed interval [0,1]. Let d be a mapping of $F \times F$ into R, the set of real numbers, defined by $d(f,g)=\int_0^1 \left|f(x)-g(x)\right| dx \ \forall f,g \in F$. Verify that d is a metric for F

[20 marks]

105. Prove that a compact set in a metric space is a closed set. **[20 marks]**

106. let $C\,[a,b]$ denote the set of all functions f on $[a,b]$ which have continuous derivatives at all points of $I = [a,b]$. For $f,g \in C\,[a,b]$ define

$d(f,g)=\left|f(a)-g(b)\right|+\sup\left\{\left|f'(x)-g'(x)\right|, x \in I\right\}$. Show that the space $(C[a,b],d)$ is a complete **[20 marks]**

107. A function f is defined in the interval (a,b) as follows:

$$f(x)=\begin{cases} q^{-2} & \text{when } x = pq^{-1} \\ q^{-3} & \text{when } x = \left(pq^{-1}\right)^{1/2} \end{cases}$$

Where p,q are relatively prime integers; $f(x)=0$, for all other values of x. Is f Rieman integrable? Justify your answer. **[20 marks]**

108. Test for uniform convergence, the series $\sum_{n=1}^{\infty} \frac{2^n x^{2n-1}}{1+x^{2n}}$ **[20 marks]**

109. Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin x \sin^{-1}\left(\sin x \sin y\right) dxdy$ **[20 marks]**

1995

110. Let K and F be nonempty disjoint closed subjects of R^2 . If K is bounded, show that there exists $\delta > 0$ such that $d(x,y) \geq \delta$ for $x \in K$ and $y \in F$ where $d(x,y)$ is the usual distance between x and y. **[20 marks]**

111. Let f be a continuous real function on R such that f maps open intervel into open intervals. Prove that f is monotonic. **[20 marks]**

112. Let $C_n \geq 0$ for all positive integers n such that is convergent. Suppose $\{S_n\}$ is a sequence of distinct points in (a,b) For $x \in [a,b]$, define $\alpha(x) = \sum C_n \{n : x > S_n\}$. Prove that α is an increasing function. If f a continuous real function on $[a,b]$, show that

$$\int_a^b f d\alpha = \sum cnf(S_n)$$ **[20 marks]**

113. Suppose f maps an open ball $U \subset R^n$ into R^m and f is differentiable on U. Suppose there exists a real number $M>0$ such that $\|f(x)\| \leq M \ \forall x \in U$. Prove that

$$|f(b) - f(a)| \leq M|b-a| \forall a, b \in U$$ **[20 marks]**

114. Find and classify the extreme values of the function $f(x,y)=x^2+y^2+x+y+xy$ **[20 marks]**

115. Suppose α is real number not equals to $n\pi$ for any integer n. Prove that

$$\int_0^\infty \int_0^\infty e^{-(x^2+2xy\cos\alpha+y^2)} dxdy = \frac{\alpha}{2\sin\alpha}$$ **[20 marks]**

1994

116. Examine the (i) absolute convergence (ii) uniform convergence of the series $(1-x)+x(1-x)+x^2(1-x)+\ldots$ in $[-c,1]$, $0 < c < 1$ **[20 marks]**

117. Prove that $S(x) = \sum \frac{1}{n^p + n^q x^2}, p > 1$ is uniformly convergent for all values of x and can be differentiable term by term if $q < 3p < 2$ **[20 marks]**

118. Let the function f be defined on [0,1] by the condition $f(x)=2rx$ when $\frac{1}{r+1} < x < \frac{1}{r}, r > 0$

Show that f is Riemann integrable in [0,1] and $\int_0^1 f(x)dx = \frac{\pi^2}{6}$ **[20 marks]**

119. By means of substitution $x+y+z=u$, $y+z=uv$, $z=uvw$ evaluate $\iiint (x+y+z)^n \, xyz \, dxdyxdz$ taken over the volume bounded by $x=0$, $y=0$, $z=0$,. $x+y+z=1$ **[20 marks]**

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120. Examine for Riemann intergrability over [0,2] of the function defined on [0,2] by

$$f(x) = \begin{cases} x + x^2 & \text{for rational valuesof } x \\ x^2 + x^3 & \text{for irrational values of } x \end{cases}$$ **[20 marks]**

121. Prove that $\int_0^\infty \frac{\sin x}{x} dx$ converges and conditionally converges. **[20 marks]**

122. Evaluate $\iiint \frac{dxdydz}{x+y+z+1}$ over the volume bounded by the coordinate planes and the plane x+y+z=1 **[20 marks]**

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123. If we metrize the space of functions continuous on [a,b] by taking $p(x,y)=\sqrt{\int_a^b \left[x(t)-y(t)\right]^2 dt}$ then show that the resulting metric space is NOT complete **[20 marks]**

124. Examine $2xyz-4zx-2yz+x^2+y^2+z^2-2x-4y-4z$ for extreme values **[20 marks]**

125. If $U_n=\frac{1+nx}{ne^{nx}}-\frac{1+(n+1)x}{(n+1)e^{(n+1)x}}, 0<x<1$ Prove that $\frac{d}{dx}\sum U_n=\sum\frac{d}{dx}U_n$. Is the series uniformly convergent in (0,1)? Justify your claim. **[20 marks]**

126. Find the upper and lower Riemann integral for the function defined in the interval [0,1] as follows $\begin{cases}\sqrt{1-x^2} & \text{when } x \text{ is rational}\\ 1-x & \text{when } x \text{ is irrational}\end{cases}$ and show that is NOT Riemann integrable in [0,1]. **[20 marks]**

127. Discuss the convergence or divergence of $\int_0^\infty \frac{x^\beta}{1+x\alpha\sin^2 x}dx,\ \alpha>\beta>0$ **[20 marks]**

128. Evaluate $\iint\sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **[20 marks]**