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MATHEMATICS

CALCULUS

Previous year Questions from **1992 To 2017**

Syllabus

Real numbers, functions of a real variable, limits, continuity, differentiability, mean-value theorem, Taylor's theorem with remainders, indeterminate forms, maxima and minima, asymptotes; Curve tracing; Functions of two or three variables: limits, continuity, partial derivatives, maxima and minima, Lagrange's method of multipliers, Jacobian.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

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2017

1. Integrate the function $f(x,y)=xy(x^2+y^2)$ over the domain
$R:\{-3\le x^2-y^2\le 3, 1\le xy\le 4\}$. **(10 marks)**

2. If $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\ne(0,0)\\ 0, & (x,y)=(0,0),\end{cases}$

Calculate $\dfrac{\partial^2 f}{\partial x\partial y}$ and $\dfrac{\partial^2 f}{\partial y\partial x}$ at (0,0) **(15 marks)**

3. Examine if the improper integral $\int_0^3 \dfrac{2x\,dx}{(1-x^2)^{2/3}}$ exists. **(10 marks)**

4. Prove that $\dfrac{\pi}{3}\le\iint_D \dfrac{dxdy}{\sqrt{x^2+(y-2)^2}}\le\pi$ where D is the unit disc. **(10 marks)**

2016

5. Evaluate: $I=\int_0^1 \sqrt[3]{x\log\left(\dfrac{1}{x}\right)}\,dx$ **(10 marks)**

6. Fidn the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions
$\dfrac{x^2}{4}+\dfrac{y^2}{5}+\dfrac{z^2}{25}=1$ and x+y–z=0 **(20 marks)**

7. Let $f(x,y)=\begin{cases}\dfrac{2x^4y-5x^2y^2+y^5}{(x^2+y^2)^2}, & (x,y)\ne(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ find a $\delta>0$ such that
$|f(x,y)-f(0,0)|<0.01$ whenever $\sqrt{x^2+y^2}<\delta$ **(15 marks)**

8. Find the surface area of the plane $x+2y+2z=12$ cut off by $x=0$, $y=0$ $x^2+y^2=16$

(15 marks)

9. Evaluate $\iint_R f(x,y)\,dxdy$, over the rectangle R = [0,1;0,1] where

$f(x,y)=\begin{cases}x+y, & if\ x^2<y<2x^2\\ 0, & elsewhere\end{cases}$ **(15 marks)**

2015

10. Evaluate the following limit $\lim_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ **(10 marks)**

11. Evaluate the following integral : $\int_{\pi/6}^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ **(10 marks)**

12. A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. **(13 marks)**

13. Which point of the sphere $x^2+y^2+z^2=1$ is at the maximum distance from the point (2,1,3) **(13 marks)**

14. Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)\,dxdy$ where R is the rhombus with successive vertices as (p,0)(2p,p),(p,2p),(0,p) **(12 marks)**

15. Evaluate $\iint_R \sqrt{|y-x^2|}\,dxdy$ where R = [–1,1;0,2] **(13 marks)**

16. For the function $f(x,y)=\begin{cases}\frac{x^2-x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0)\\ 0 \quad , & (x,y)=(0,0)\end{cases}$. Examine the continuity and differentiability. **(12 marks)**

2014

17. Prove that between two real roots $e^x cosx+1=0$, a real root of $e^x sinx+1=0$ lies. **(10 marks)**

18. Evaluate $\int_0^1 \frac{\log_e(1+x)}{1+x^2}dx.$ **(10 marks)**

19. By using the transformation $x+y=u,\ y=uv$ evaluate the integral $\iint\{xy(1-x-y)\}^{\frac{1}{2}}\,dxdy$ taken over the area enclosed by the straight lines $x=0$, $y=0$ and $x+y=1$. **(15 marks)**

20. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. **(15 marks)**

21. Find the maximum or minimum values of $x^2+y^2+z^2$ subject to the condition $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$ interpret result geometrically **(20 marks)**

2013

22. Evaluate $\int_0^1 \left(2x\sin\frac{1}{x}-\cos\frac{1}{x}dx\right)$ **(10 marks)**

23. Using Lagrange's multiplier method find the shortest distance between the line $y=10-2x$ and the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ **(20 marks)**

24. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function $f(x,y)=\begin{cases}\frac{xy^3}{x+y^2}, (x,y)\neq(0,0)\\ 0, \quad (x,y)=(0,0)\end{cases}$ Also discuss the continuity of f_{xy} and f_{yx} at (0,0). **(15 marks)**

25. Evaluate $\iint_D xydA$ where D is the region bounded by the line $y=x-1$ and the parabola $y^2=2x+6$. **(15 marks)**

2012

26. Define a function f of two real variables in the plane by

$$f(x,y)=\begin{cases}\frac{x^3\cos\frac{1}{y}+y^3\cos\frac{1}{x}}{x^2+y^2} \text{ for } x,y\neq 0\\ 0, \text{ otherwise}\end{cases}$$

Check the continuity and differentiability of f at (0,0). **(12 marks)**

27. Let p and q be positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$ show that for real numbers $a,b\geq 0\, ab\leq\frac{a^p}{p}+\frac{b^q}{q}$. **(12 marks)**

28. Find the point of local extrema and saddle points of the function f for two variable defined by $f(x,y)=x^3+y^3-63(x+y)+12xy$ **(20 marks)**

29. Defined a sequence S_n of real numbers by $S_n=\sum_{i=1}^{n}\frac{(\log(n+i)-\log n)^2}{n+1}$ does $\lim_{n\to\infty} S_n$ exist? If so compute the value of this limit and justify your answer **(20 marks)**

30. Find all the real values of p and q so that the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$ converges

(20 marks)

2011

31. Find $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3+y^3}$ if exists **(10 marks)**

32. Let f be a function defined on *IR* such that $f(0) = -3$ and $f(x)$ £ 5 for all values of x in *IR*. How large can $f(2)$ possibly be? **(10 marks)**

33. Evaluate:

(i) $\lim_{x\to 2} f(x)$ Where $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ \pi \quad , & x = 2 \end{cases}$

(ii) $\int_0^1 \ln x dx$ **(10 marks)**

2010

34. A twice differentiable function $f(x)$ is such that $f(a) = 0 = f(b)$ and $f(c) > 0$ for $a<c<b$ prove that there be is at least one point $\xi, a < \xi < b$ for which $f''(\xi) < 0$ **(12 marks)**

35. Dose the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}$ exist if so find its value **(12 marks)**

36. Show that a box (rectangular parallelepiped) of maximum volume V with prescribed surface area is a cube. **(20 marks)**

37. Let D be the region determine by the inequaliteis $x>0$, $y>0$, $z<8$ and $z>x^2+y^2$ compute $\iiint_D 2x dx dy dz$. **(20 marks)**

38. If $f(x,y)$ is a homogeneous function of degree n in x and y, and has continuous first and second order partial derivatives then show that

(i) $x\frac{\partial^2 f}{\partial x} + y\frac{\partial^2 f}{\partial y} = nf$

(ii) $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$ **(20 marks)**

2009

39. Suppose the f'' is continuous on [1,2] and that f has three zeroes in the interval (1,2) show that f'' has least one zero in the interval (1,2). **(12 marks)**

40. If f is the derivative of some function defined on [a,b] prove that there exists a number h Î $[a,b]$ such that $\int_a^b f(t)dt = f(\eta)(b-a)$ **(12 marks)**

41. If $x = 3 \pm 0.01$ and $y = 4 \pm 0.01$ with approximately what accuracy can you calculate thepolar coordinate r and q of the point $P(x,y)$? Express you estimates as percentage changes of the value that r and q have at the point (3,4)

(20 marks)

42. A space probe in the shape of the elliposoid $4x^2+y^2+4z^2=16$ enters the earth atmosphere and its surface beings to heat. After one hour, the temperature at the point (x,y,z) on the probe surface is given by $T(x,y,z) = 8x^2+4yz-16z+600$ Find the hottest point on the probe surface. **(20 marks)**

43. Evaluate $I = \iint_s xdydz + dzdx + xz^2dxdy$ where S is the outer side of the part of the sphere $x^2+y^2+z^2=1$ in the first octant. **(20 marks)**

2008

44. Find the value of $\lim_{x\to 1} ln(1-x)\cot\frac{\pi x}{2}$. **(12 marks)**

45. Evaluate $\int_0^1 (x\, ln\, x)^3\, dx$. **(12 marks)**

46. Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2+4xy+6y^2=140$ **(20 marks)**

47. Evaluate the double integral $\int_y^a \frac{xdxdy}{x^2+y^2}$ by changing the order of integration

(20 marks)

48. Obtain the volume bounded by the elliptic paraboloid given by the equations $z=x^2+9y^2$ & $z=18-x^2-9y^2$ **(20 marks)**

2007

49. Let $l(x)(x \in (-\pi,\pi))$ be defined by $f(x) = \sin|x|$ is f continuous on (–p,p) if it is continuous then is it differentiable on (–p,p)? **(12 marks)**

50. A figure bounded by one arch of a cycloid $x = a(t-sint)$, $y=a(1-cost)$, t Î [0,2p] and the x–axis is revolved about the x-axis. Find the volume of the solid of revolution **(12 marks)**

51. Find a rectangular parallelepiped of greatest volume for a give total surface area S using Lagrange's method of multipliers **(20 marks)**

52. Prove that if $z = \phi(y+ax)+\psi(y-ax)$ then $a^2\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$ for any twice differentiable f and y a is a constant. **(15 marks)**

53. Show that $e^{-x}x^{n}$ is bounded on [0,¥) for all positive integral values of n using this result show that $\int_{0}^{\infty} e^{-x}x^{n}\,dx$ axis's. **(15 marks)**

2006

54. Find a and b so that $f'(2)$ exists where $f(x)=\begin{cases}\frac{1}{|x|}, & if\ |x|>2\\ a+bx^{2} & if\ |x|\le 2\end{cases}$ **(12 marks)**

55. Express $\int_{0}^{1} x^{m}\left(1-x^{n}\right)^{p} dx$ in terms of Gamma function and hence evaluate the integral $\int_{0}^{1} x^{6}\sqrt{\left(1-x^{2}\right)}dx$ **(12 marks)**

56. Find the values of a and b such that $\lim_{x\to 0}\frac{a\sin^{2}x \times b\log\cos x}{x^{4}}=\frac{1}{2}$. **(15 marks)**

57. If $z=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$ show that $x^{2}\frac{\partial^{2}z}{dx^{2}}+2xy\frac{\partial^{2}z}{\partial x^{2}}+y^{2}\frac{\partial^{2}z}{\partial y^{2}}=0$. **(15 marks)**

58. Change the order of integration in $\int_{x}^{\infty}\frac{e^{-y}}{y}dydx$ and hence evaluate it. **(15 marks)**

59. Find the volume of the uniform ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$ **(15 marks)**

2005

60. Show that the function given below is not continuous at the origin

$$f(x,y)=\begin{cases}0 & if\ \ xy=0\\ 1 & if\ \ xy\neq 0\end{cases}$$ **(12 marks)**

61. Let R^{2}®R be defined as $f(x,y)=\frac{xy}{\sqrt{x^{2}+y^{2}}},(x,y)\neq(0,0)\,f(0,0)=0$ prove that f_{x} and f_{y} exist at (0,0) but f is not differentiable at (0,0). **(12 marks)**

62. If $u=x+y+z$, $uv=y+z$ and $uvw=z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ **(15 marks)**

63. Evaluate $\int_{0}^{1}\frac{x^{m-1}+n-1}{(1+x)^{m+n}}dx$ in terms of Beta function. **(15 marks)**

64. Evaluate $\iiint_v z\,dv$ where V the volume is bounded below by the cone $x^2+y^2=z^2$ and above by the sphere $x^2+y^2+z^2=1$ lying on the positive side of the y-axis. **(15 marks)**

65. Find the x-coordinate of the center of gravity of the solid lying inside the cylinder $x^2+y^2=2ax$ between the plane $z=0$ and the paraboloid $x^2+y^2=az$. **(15 marks)**

2004

66. Prove that the function f defined on [0,4] $f(x)=[x]$ greatest integer $\leq x, x \in [0,4]$ is integrable on [0,4] and that $\int_0^4 f(x)\,dx=6$ **(12 marks)**

67. Show that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}\; x>0$. **(12 marks)**

68. Let the roots of the equation in $\lambda(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ be u,v,w proving that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$. **(15 marks)**

69. Prove that an equation of the form $x^n=\alpha$ where $\frac{ne}{N}$ and $\alpha>0$ is a real number has a positive root **(15 marks)**

70. Prove that $\int\frac{x^2+y^2}{p}dx=\frac{\pi ab}{4}\left[4+\left(a^2+b^2\right)\left(a^{-2}+b^{-2}\right)\right]$ when the integral is taken round the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and p is the length of three perpendicular from the center to the tangent **(15 marks)**

71. If the function f is defined by $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$ then show that f possesses both the partial derivatives at (0,0) but it is not continuous thereat.

(15 marks)

2003

72. Let f be a real function defined as follow: $\begin{cases} f(x) = x, - \leq x < 1 \\ f(x+2) = x, \forall x \in R \end{cases}$ Show that f is discontinuous at every odd integer **(12 marks)**

73. For all real numbers x, $f(x)$ is given as $f(x) = \begin{cases} e^x + a\sin x, & x < 0 \\ b(x-1)^2 + x - 2, & x \geq 0 \end{cases}$ Find values of a and b for which is differentiable at $x=0$ **(12 marks)**

74. A rectangular box open at the top is to have a volume of 4 Using Lagrange's method of multipliers find the dimension of the box so that the material of a given type required to construct it may be least. **(15 marks)**

75. Test the convergent of the integrals (i) $\int_0^a \frac{dx}{x^{\frac{1}{3}}(1+x^2)}$ (ii) $\int_0^\infty \frac{\sin^2 x}{x^2} dx$ **(15 marks)**

76. Evaluate the integral $\int_0^a \int_{\frac{y^2}{a}}^{y} \frac{y\,dx\,dy}{(a-x)\sqrt{ax-y^2}}$ **(marks)**

77. Find the volume generated by revolving by the real bounded by the curves $(x^2+4a^2)y = 8a^3, 2v = x$ and $x = 0$, about the y-axis. **(15 marks)**

2002

78. Show that $\frac{b-a}{\sqrt{1-a^2}} \leq \sin^{-1} b - \sin^{-1} a \leq \frac{b-a}{\sqrt{1-b^2}}$ for $0<a<b<1$. **(12 marks)**

79. Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy = \frac{\pi}{4}$ **(12 marks)**

80. Let $f(x) = \begin{cases} x^p \sin\frac{1}{x}, & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$ Obtain condition on p such that (i) f is continuous at $x=0$ and (ii) f is differentiable at $x=0$ **(15 marks)**

81. Consider the set of triangle having a given base and a given vertex angle show that the triangle having the maximum area will be isosceles **(15 marks)**

82. If the roots of the equation $(\lambda-u)^3 + (\lambda-v)^3 + (\lambda-w)^3 = 0$ in l are x,y,z show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$. **(15 marks)**

83. Find the center of gravity of the region bounded by the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$ and both axes in the first quadrant the density being $r = kxy$ where k is constant. **(15 marks)**

2001

84. Let f be defined on IR by setting $f(x)=x$ if x is rational and $f(x)=1-x$ if is irrational show that f is continuous at $x=\frac{1}{2}$ but is discontinuous at every other point. **(12 marks)**

85. Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}dx.$ **(12 marks)**

86. Find the equation of the cubic curve which has the same asymptotes as $2x(y-3)^2=3y(x-1)^2$ and which touches the x – axis at the origin and passes though the point (1,1). **(15 marks)**

87. Find the maximum and minimum radii vectors of the section of the surface $\left(x^2+y^2+z^2\right)=a^2x^2+b^2y^2+c^2z^2$ by the plane $lx+my+nz=0$ **(15 marks)**

88. Evaluate $\iiint(x+y+z+1)^2\,dxdydz$ over the region defined by $x\geq 0, y\geq 0, z\geq 0, x+y+z\leq 1$ **(15 marks)**

89. Find the volume of the solid generated by revolving the cardioid $r=a(1-\cos\theta)$ about the initial line **(15 marks)**

2000

90. Use the mean value theorem to prove that $\frac{2}{7}<\log 1.4<\frac{2}{5}$. **(12 marks)**

91. Show that $\iint x^{2t-1}y^{2m-1}dxdy=\frac{1}{4}r^{2(l+m)}\frac{\frac{\Gamma}{\Gamma m}}{\Gamma(l+m+1)}$ for all positive values of x and y lying inside the circle $x^2+y^2=r^2$. **(12 marks)**

92. Find the center of gravity of the positive octant of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ if the density varies as xyz **(15 marks)**

93. Let $f(x)=\begin{cases}0, x \text{ is irrational}\\ 1, x \text{ is rational}\end{cases}$ show that *f* is not Riemann integrable on [a,b]

(15 marks)

94. Show that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right)=(-1)^n\frac{n!}{x^{n+1}}\left(\log x-1-\frac{1}{2}-\frac{1}{3}\ldots\frac{1}{n}\right)$ **(15 marks)**

95. Find constant *a* and *b* for which $F(a,b)=\int_0^{\pi}\left\{\sin x-\left(ax^2+bx\right)\right\}^2 dx$ is a minimum

(15 marks)

1999

96. If *f* is Riemann integral over every interval of finite length and $f(x+y)=f(x)+f(y)$ for every pair of real numbers *x* and *y* show that *f(x)=cx* where *c* = *f*(1) **(20 marks)**

97. Show that the area bounded by cissoids $x=a\sin^2 t$, $y=a\frac{\sin^3 t}{\cos t}$ and its asymptote is $\frac{3\pi a^2}{4}$ **(20 marks)**

98. Show that $\iint x^{m-1}y^{n-1}\,dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{a^m b^n}{4}\frac{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{m}{2}+\frac{n}{2}+1\right)}$$ **(20 marks)**

1998

99. Find the asymptotes of the curve (2*x*–3*y*+1)2+(*x*+*y*)–8*x*+2*y*–9=0 and show that they intersect the curve again in three points which lie on a straight line.

(10 marks)

100. A thin closed rectangular box is to have one edge n times the length of another edge and the volume of the box is given to be v. Prove that the least surface s is given by $ns^3=54(n+1)^2v^2$. **(10 marks)**

101. If *x*+*y*=1, Prove that

$$\frac{d^n}{dx^n}\left(x^n y^n\right)=n!\left[y^n-\binom{n}{1}^2 y^{n-1}x+\binom{n}{2}^2 y^{n-2}x^2+\ldots+(-1)^n x^n\right]$$ **(10 marks)**

102. Show that $\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}dx = B(p,q)$

103. Show that $\iiint \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}} = \frac{\pi^2}{8}$

1997

104. Suppose $f(x)=17x^{12}-124x^9+16x^3-129x^2+x-1$ determine $\frac{d}{dx}(f^{-1})$ if $x=-1$ it exists. **(20 marks)**

105. Prove that the volume of the greatest parallelepiped that can be inscribe in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1\frac{8abc}{3\sqrt{3}}$ **(20 marks)**

106. Show that the asymptotes of the cut the curve $(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^2+zy^3-x^2+3xy-1=0$ again in eight points which lie on a circle of radius 1. **(20 marks)**

107. An area bounded by a quadrant of a circle of radius a and the tangent at its extremities revolve about one of the tangent Find the volume so generated. **(20 marks)**

108. Show how the canges of order in the integral $\int_0^\infty \int_0^\infty e^{-xy}\sin x dxdy$ leads to the evalutation of $\int_0^\infty \frac{\sin}{x}dx$ hence evaluate it. **(20 marks)**

109. Show that in $\sqrt{n}\sqrt{n+\frac{1}{2}} = \frac{\sqrt{\pi}}{2^{2n-1}}$ where $n>0$ and $\sqrt{n}$ denote gamma function. **(20 marks)**

1996

110. Find the asymptotes of all curves $4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$ and show that they pass thought the point of intersection of the curve with the ellipse $x^2+4y^2=4$. **(20 marks)**

111. Show that any continuous function defined for all real x and satisfying the equation $f(x)=f(2x+1)$ for all x must be a constant function. **(20 marks)**

112. Show that the maximum and minimum of the radii vectors of the section of the surface $\left(x^2+y^2+z^2\right)^2 \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ by the plane $lx+my+vz=0$ are given by the equation $\frac{a^2\lambda^2}{I-a^2r^2}+\frac{b^2\mu^2}{I-b^2r^2}+\frac{a^2v^2}{I-c^2r^2}=0$. **(20 marks)**

113. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$ **(20 marks)**

114. Evaluate $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$. **(20 marks)**

115. The area cut off from the parabola $y^2=4ax$ by chord joining the vertex to an end of the latus rectum is rotated though four right angle about the chord. Find the volume of the solid so formed. **(20 marks)**

1995

116. If g is the inverse of f and $f'(x)=\frac{1}{1+x^3}$ prove that $g(x)=1+\left[g(x)\right]^3$ **(20 marks)**

117. Taking the nth derivative of $\left(x^n\right)^2$ in two different ways show that

$1+\frac{n^2}{I^2}+\frac{n^2}{I^2 2^2}+\frac{n^2(n-1)^2}{I^2,2^2,3^2}+....$ to $(n+1)term=\frac{(2n)!}{(n!)^2}$ **(20 marks)**

118. Let $f(x,y)$ which possesses continuous partial derivatives of second order be a homogeneous functionf of x and y of degree n prove that

$x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f$. **(20 marks)**

119. Find the area bounded by the curve $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)=\frac{x^2}{4}-\frac{y^2}{9}$. **(20 marks)**

120. Let $f(x), x\geq 1$ be such that the area bounded by the curve $y=f(x)$ and the lines $x=1$, $x=b$ is equal to $\sqrt{1+b^2}-\sqrt{2}$ for all b ³ 1 Does f attain its minimum? If so what is its values? **(20 marks)**

121. Show that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)...\Gamma\left(\frac{n-1}{n}\right)=\frac{(2\pi)}{\sqrt{n}}\frac{n-1}{2}$. **(20 marks)**

1994

122. $f(x)$ is defined as follows : $f(x)=\begin{vmatrix} \frac{1}{2}\left(b^2-a^2\right) & of\ \ 0<x\le a \\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^2}{3x} & of\ a<x\le b \\ \frac{1}{3}\frac{b^3-a^3}{x} & of\ \ x>b \end{vmatrix}$ Prove that $f(x)$ and $f'(x)$ are continuous but $f''(x)$ is discontinuous. **(20 marks)**

123. If a and b lie between the least and greatest values of a,b,c prove that

$$\begin{vmatrix} f(a) & f(b) & f(c) \\ \phi(a) & \phi(b) & \phi(c) \\ \psi(a) & \psi(b) & \psi(c) \end{vmatrix} = K \begin{vmatrix} f(a) & f'(\alpha) & f''(\beta) \\ \phi(a) & \phi'(\alpha) & \phi''(\beta) \\ \psi(x) & \psi'(\alpha) & \psi''(\beta) \end{vmatrix}$$ Where $K\frac{1}{2}(b-c)(c-a)(a-b)$

(20 marks)

124. Prove that all rectangular parallelpipeds of same volume, the cube has least surface **(20 marks)**

125. Show that means of beta function that $\int_f^z \frac{dx}{(z-x)^{1-a}(x-t)^a} = \frac{\pi}{\sin \pi\alpha}(0<\alpha<1$.

(20 marks)

126. Prove that the value of $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the co-ordinate planes and the plane $x+y+z=1$ is $\frac{1}{2}\left(\log 2-\frac{5}{8}\right)$. **(20 marks)**

127. The sphere $x^2+y^2+z^2=a^2$ is pierced by the cylinder $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right)$ prove that the volume of the sphere that lies inside the cylinder is $\frac{8a^3}{3}\left[\frac{\pi}{4}+\frac{5}{3}=\frac{4\sqrt{2}}{3}\right]$

(20 marks)

1993

128. Prove that $f(x)=x^2\sin\frac{1}{x}, x\ne 0$ and $f(x)=0$ for $x=0$ is continuous and differentiable at $x=0$ but its derivative is not continuous there. **(20 marks)**

129. If $f((x), f(x), y(x)$ have derivative when $a \le x \le b$ show that there is a values c of x lying between a and b such that $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f(c) & \phi(c) & \psi(c) \end{vmatrix} = 0$ **(20 marks)**

130. Find the triangle of maximum area which can be inscribed in a circle **(20 marks)**

131. Prove that $\int_0^\infty e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} (a > 0)$ deduce that $\int_0^\infty x^{2n} e^{-x^2} dx = \frac{\sqrt{\pi}}{2^{n+1}} [1.3.5...(2n-1)]$

(20 marks)

132. Defined Gamma function and prove that $\lceil n \left| \left(n + \frac{1}{2} \right) \right. = \frac{\sqrt{\pi}}{2^{2n-1}} \lceil 2n$ **(20 marks)**

133. Show that volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=ax$ is $\frac{2a^2}{9}(3\pi - 4)$. **(20 marks)**

1992

134. If $y = e^{ax} \cos bx$ prove that $y_2 - 2ay_1 + (a^2+b^2)y = 0$ and hence expand $e^{2x} \cos bx$ in powers of x Deduce the expansion of e^{ax} and $cosbx$. **(20 marks)**

135. If $x = r\, sinq\, cosf$, $y = r\, sinq\, sinf$, $z = rcosq$ then prove that $dx^2+dy^2+dz^2 = dr^2+r^2dq^{\,2}+ r^2sin^2q\; df^{\,2}$. **(20 marks)**

136. Find the dimension of the rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ that has greatest volume **(20 marks)**

137. Prove that the volume enclosed by the cylinders $x^2+y^2=2ax$, $z^2=2$ axis $128a^3/15$ **(20 marks)**

138. Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the x-axis **(20 marks)**

139. Evaluate the following integral in terms of Gamma function $\int_{-1}^{+1} (1+x)^p (1-x)^q dx, [p > -1, q > -1]$ and prove that $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) = \frac{2}{\sqrt{3}}\pi$ **(20 marks)**