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MATHEMATICS

VECTOR ANALYSIS

Previous year Questions from **1992 To 2017**

Syllabus

Scalar and vector fields, differentiation of vector field of a scalar variable; Gradient, divergence and curl in cartesian and cylindrical coordinates; Higher order derivatives; Vector identities and vector equations. Application to geometry: Curves in space, Curvature and torsion; Serret-Frenet's formulae. Gauss and Stokes' theorems, Green's identities.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

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2017

1. Suppose U and W are distinct four dimensional subspaces of a vector space V, where dim V=6. Find the possible dimensions of subspace $U \cap W$. **(10 Marks)**

2. Evaluate the integral : $\iint_S \overline{F}.\hat{n}ds$ where $\overline{F} = 3xy^2\hat{i} + (yx^2 - y^3)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \le 4, -3 \le x \le 3$, using divergence theorem. **(9 Marks)**

3. Using Green's theorem, evaluate the $\int_C F(\overline{r}).d\overline{r}$ counterclockwise where $F(\overline{r}) = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$ and $d\overline{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \{(x, y) | 1 \le y \le 2 - x^2\}$. **(8 Marks)**

2016

4. Prove that the vector $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}, \vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle find the length of the medians of the triangle **(10 Marks)**

5. Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1)=0$ **(10 Marks)**

6. Prove that $\oint_C f d\vec{r} = \iint_S d\vec{S} \times \nabla f$ **(10 Marks)**

7. For the of cardioid $r = a(1+\cos\theta)$ show that the square of the radius of curvature at any point (r, θ) is proportion to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. **(15 Marks)**

2015

8. Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z=x^2+y^2-3$ at (2,–1,2) **(10 Marks)**

9. A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. **(12 Marks)**

10. Evaluate $\int_C e^{-x}(\sin y dx + \cos y dy)$, Where C is the rectangle with vertices (0,0) (π,0), ($\pi, \frac{\pi}{2}$), (0, $\frac{\pi}{2}$) **(12 Marks)**

2014

11. Find the curvature vector at any point of the curve $\overline{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude also. **(10 Marks)**

12. Evaluate by Stoke's theorem $\int_r (ydx + zdy + xdz)$, where Γ is the curve given by $x^2+y^2+z^2-2ax-2ay=0$, $x+y=2a$ starting from $(2a,0,0)$ and then going below the z-plane **(20 Marks)**

2013

13. Show the curve $\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. **(10 Marks)**

14. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplance operator. **(10 Marks)**

15. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$ **(10 Marks)**

16. By using Divergence Theorem of Gauss, evaluate the surface integral $\iint (a^2x^2 + b^2y^2 + c^2z^2)^{-\frac{1}{2}}$ dS. where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$, a,b and c being all positive constants. **(15 Marks)**

17. Use Stroke's theorem to evaluate the line integral $\int_c (-y^2dx + x^2dy - z^3dz)$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$ **(15 Marks)**

2012

18. If $\vec{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\vec{k}, \vec{B} = 2z\hat{i} + y\hat{j} - x^2\vec{k}$ find the value of $\frac{\partial^2}{\partial x \partial y}(\vec{A} + \vec{B})$ at $(1,0,-2)$ **(12 Marks)**

19. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Show that the curvature and torsion are equal for this curve. **(20 Marks)**

20. Verify Green's theorem in the plane for $\oint_C [xy + y^2dx + x^2dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ **(20 Marks)**

21. If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate $\iint_S (\vec{\nabla} \times \vec{F}).\vec{n}\vec{ds}$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy - plane. **(20 Marks)**

2011

22. For two vectors $\vec{a}$ and $\vec{b}$ give respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and

$\vec{b} = \sin 5t\hat{i} - \cos t\hat{j}$ determine:(i) $\frac{d}{dt}\left(\vec{a}.\vec{b}\right)$ and (ii) $\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **(10 Marks)**

23. If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f}$ =$grad v$, find the value of $\vec{f}\ curl\ \vec{f}$ **(10 Marks)**

24. Examine whether the vectors $\nabla u, \nabla v, \nabla w$ are copalanar, where u,v and w are the scalar functions defined by:
$u=x+y+z$,
$v=x^2+y^2+z^2$
and $w=yz+zx+xy$ **(15 Marks)**

25. If $u = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ calculate double integral $\iint\left(\nabla\times\vec{u}\right)d\vec{s}$ over the hemisphere given by $x^2+y^2+z^2$=a^2, z $\geq$ 0 **(15 Marks)**

26. If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector. $r^n\vec{r}$
(i) irrotational, (ii) solenoidal **(15 Marks)**

27. Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube 0 $\leq x, y, z \leq$ 1. **(15 Marks)**

2010

28. Find the directional derivative of $f(x,y)=x^2y^3+xy$ at the point (2,1) in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis. **(12 Marks)**

29. Show that the vector field defined by the vector function $\vec{v} = xyz\left(yz\vec{i} + xy\vec{j} + xy\vec{k}\right)$ is conservative. **(12 Marks)**

30. Prove that $div(f\vec{V}) = f\left(div\vec{V}\right) + \left(grad.f\right)\vec{V}$ where f is a scalar function. **(20 Marks)**

31. Use the divergence theorem to evaluate $\iint_s \vec{V}\vec{n}dA$ where $\vec{V} = x^2z\vec{i} + y\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4y$.
(20 Marks)

32. Verify Green's theorem for $e^{-x}sinydx+e^{-x}cosydy$ the path of integration being the boundary of the sqaure whose vertices are (0,0), $(\frac{\pi}{2},0)$, $(\frac{\pi}{2},\frac{\pi}{2})$, and $(0,\frac{\pi}{2})$

(20 Marks)

2009

33. Show that $div(grad r^n)=n(n+1)r^{n-2}$ where $r = \sqrt{x^2+y^2+z^2}$. **(12 Marks)**

34. Find the directional derivative of (i) $4xz^3-3x^2y^2z^2$ (i) at (2,–1,1) along z-axis (ii) $-x^2yz+4xz^2$ at (1,–2,1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$. **(6+6=12 Marks)**

35. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, z=0 under the field of force given by $\overline{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$. **(20 Marks)**

36. Using divergence theorem, evaluate $\iint_s \overline{A}.d\overline{S}$ where $\overline{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ **(20 Marks)**

37. Find the value of $\iint_s (\vec{\nabla}\times\vec{f}).d\vec{s}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane z=0, when $\overline{F}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$ **(20 Marks)**

2008

38. Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1,–1,2). **(12 Marks)**

39. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field (1,–2,1) to (3,1,4). **(12 Marks)**

40. Prove that $\nabla^2 f(x)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=(x^2+y^2+z^2)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r)=0$. **(15 Marks)**

41. Show that for the space curve $x=t$, $y=t^2$, z=$\frac{2}{3}t^3$ the curvature and torsion are same at every point. **(15 Marks)**

42. Evaluate $\int_c \vec{A}d\vec{r}$ along the curve $x^2+y^2=1$, z=1 from (0,1,1) to (1,0,1) if $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. **(15 Marks)**

43. Evaluate $\iint_s \vec{F}\hat{n}ds$ where $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$, $\iint_s \vec{F}\hat{n}ds$ and S is the surface of the cylinder bounded by $x^2+y^2=4$, $z=0$ and z=3 **(15 Marks)**

2007

44. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$ determined grad (r^{-1}) in terms of $\hat{r}$ and r. **(12 Marks)**

45. Find the curvature and torsion at any point of the curve $x = a\cos 2t.\ y = a\sin 2t,\ z = 2a\sin t.$ **(12 Marks)**

46. For any constant vector, show that the vector $\vec{a}$ represented by curl $(\vec{a} \times \vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x,y,z) measured from the origin. **(15 Marks)**

47. If $\vec{r} = x\hat{i} + y\hat{j} + x\hat{k}$ find the value(s) of n in order that $r^n\vec{r}$ may be
(i) solenoidal (ii) irrotational **(15 Marks)**

48. Determine $\int_c (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2+(y-a)^2+z^2=2a^2$, $x+y=2a$ that starts from the point $(2a,0,0)$ goes at first below the z-plane **(15 Marks)**

2006

49. Find the values of constants a,b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^2$ at the point (1,2,–1) has maximum magnitude 64 in the direction parallel to z-axis. **(12 Marks)**

50. If $\overline{A} = 2\overline{i} + \overline{K}, \overline{B} = \overline{i} + \overline{j} + \overline{k}, \overline{C} = 4\overline{i} - 3\overline{j} - 7\overline{K}$ determine a vector $\overline{R}$ satisfying the vector equation $\overline{R} \times \overline{B} = \overline{C} \times \overline{B}\ \&\ \overline{R}.\overline{A} = 0$ **(15 Marks)**

51. Prove that $r^n\overline{r}$ is an irrotational vector for any value of n but is solenoidal only if $n+3=0$ **(15 Marks)**

52. If the unit tangent vector $\overline{t}$ and binormal $\overline{b}$ make angles ϕ and ϕ respectively with a constant unit vector $\overline{a}$ prove that $\frac{\sin\theta}{\sin\phi}.\frac{d\theta}{d\phi} = -\frac{k}{\tau}$. **(15 Marks)**

53. Verify Stoke's theorem for the function $\overline{F} = x^2\overline{i} - xy\overline{j}$ integrated round the sqaure in the plane $z=0$ and bounded by the lines $x=0$, $y=0$, $x=a$ and $y=a$, $a>0$. **(15 Marks)**

2005

54. Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}(\overrightarrow{AB} \times \overrightarrow{AC})\overrightarrow{AD}$ Hence find the volume of the tetrahedron with vertices (2,2,2), (2,0,0), (0,2,0) and (0,0,2) **(12 Marks)**

55. Prove that the curl of a vector field is independent of the choice of coordinates **(12 Marks)**

56. The parametric equation of a circular helix is $r = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$ where c is a constant and u is a parameter. Find the unit trangent vector $\hat{i}$ at the point u and the arc length measured form u=0 Also find $\frac{d\hat{i}}{ds}$ where S is the arc length. **(15 Marks)**

57. Show that curl $\left(k \times grad\frac{1}{r}\right) + grad\left(k.grad\frac{1}{r}\right) = 0$ where r is the distance from the origin and K is the cunit vector in the direction OZ **(15 Marks)**

58. Find the curvature and the torsion of the space curve **(15 Marks)**

59. Evaluate $\iint_s x^3 dydz + x^2 ydzdx + x^2 zdxdy$ by Gauss divergence theorem, where S is the surface of the cylinder $x^2+y^2=a^2$ bounded by z=0 and $x=b$ **(15 Marks)**

2004

60. Show that if $\overline{A}$ and $\overline{B}$ are irrotational, then $\overline{A} \times \overline{B}$ is solenodial. **(12 Marks)**

61. Show that the Frenet-Serret formuale can be written in the form

$$\frac{d\overline{T}}{ds} = \overline{\omega} \times \overline{T}, \frac{d\overline{N}}{ds} = \overline{\omega} \times \overline{N} \ \& \ \frac{d\overline{B}}{dx} = \overline{\omega} \times \overline{B}$$, where $\overline{\omega} = \tau\overline{T} + k\overline{B}$. **(12 Marks)**

62. Prove the identity $\nabla\left(\overline{A}.\overline{B}\right) = \left(\overline{B}.\nabla\right)\overline{A} + \left(\overline{A}.\nabla\right)\overline{B} + \overline{B} \times \left(\nabla \times \overline{A}\right) + \overline{A} \times \left(\nabla \times \overline{B}\right)$ **(15 Marks)**

63. Derive the identity $\iiint_v \left(\phi\nabla^2\psi - \psi D^2\phi\right) dV = \iint_s \left(\phi\nabla\psi - \psi\nabla\phi\right)\hat{n}dS$ Where V is the volume bounded by the closed surface S. **(15 Marks)**

64. Verify Stoke's theorem for $\hat{f} = (2x - y)\hat{i} - yz^2\hat{j} - z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. **(15 Marks)**

2003

65. Show that if a' b' and c' are the reciprocals of the non-coplanar vectors, a, b and c, then any vector r may be expressed as $r = (r.a')a + (r.b')b + (r.c')c$. **(12 Marks)**

66. Prove that the divergence of a vector field is invariant $w.r.$ to co-ordinate transforma tions. **(12 Marks)**

67. Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions. **(15 Marks)**

68. Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A \times (\nabla \times A)$ where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ **(15 Marks)**

69. Find the radii of curvature and torsion at a point of intersection of the surface

$x^2-y^2=c^2,\ y = x\tanh\left(\frac{z}{c}\right)$. **(15 Marks)**

70. Evaluate $\iint_s curlA.ds$ Where S is the open surface $x^2+y^2-4x+4z=0$, $z \geq 0$ and

$A=\left(y^2+z^2-x^2\right)\hat{i}+\left(2z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-3z^2\right)\hat{k}$. **(15 Marks)**

2002

71. Let $\overline{R}$ be the unit vector along the vector $\overline{r}(t)$. Show that $\overline{R}\times\frac{\overline{dR}}{dt}=\frac{\overline{r}}{r^2}\times\frac{\overline{dr}}{dt}$ where

$r=\left|\overline{r}\right|$ **(12 Marks)**

72. Find the curvature k for the space curve x=acosθ, y=asinθ, z=aθtanα **(15 Marks)**

73. Show that $\left(curl\overline{v}\right)=grad\left(div\overline{v}\right)-\nabla^2\overline{v}$ **(15 Marks)**

74. Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it. Show that

$\iint_s\left(fgradf\right).\hat{n}ds=\iiint_v\left[\left|gradf\right|^2+f\nabla^2 f\right]dv$ Hence $\iint_s\left(fgradf\right).\hat{n}ds$ or otherwise evaluate

for $f=2x+y+2z$ over $s\equiv x^2+y^2+z^2=4$ **(15 Marks)**

75. Find the values of constants a,b and c such that the maximum value of directionial derivative of $f=axy^2+byz+cx^2z^2$ at (1,–1,1) is in the direction parallel to y-axis and has magnitude 6. **(15 Marks)**

2001

76. Find the length of the arc of the twisted curve r = $(3t.3t^2,2t^3)$ from the point t=0 to the point t=1. Find also the unit tangent t, unit normal n and the unit binormal b at t=1 **(12 Marks)**

77. Show that $curl\frac{a\times r}{r^3}=-\frac{a}{r^3}+\frac{3r}{r^5}(a.r)$ where a is constant vector. **(12 Marks)**

78. Find the directional derivative of $f=x^2yz^3$ along $x=e^{-t}$, y=1+2sin$t$, $z=t-cost$ at t=0 **(15 Marks)**

79. Show that the vector field defined by $F=2xyz^3i+x^2z^3j+3x^2yz^2k$ is irrotataional. Find also the scalar u such that $F=grad\,u$ **(15 Marks)**

80. Verify Gauss' divergence theorem of A=$(4x,-2y^2,z^2)$ taken over the region bounded by $x^2+y^2=4$, z=0 and z=3 **(15 Marks)**

2000

81. In What direction from the point (–1,1,1) is the directional derivative f=x^2yz^3 a maximum? Compute its magnitude **(12 Marks)**

82. (i)Show that the covariant derivatives of the fundamental metric tensors $g_{ij}, g^{ij}, \delta^i_j$, Vanish
(ii) Show that simultaneity is relative in special relativity theory. **(6+6=12 Marks)**

83. Show that
(i) $(A+B).(B+C)\times(C+A)=2A.B\times C$
(ii) $\nabla\times(A\times B)=(B.\nabla)A-B(\nabla.A)-(A.\nabla)B+A(\nabla.B)$ **(7+8=15 Marks)**

84. Evaluate $\iint_s F.Nds$ Where $F=2xyi+yz^2j+xzk$ and S is the surface of the parallelepiped bounded by $x=0$, $y=0$, $z=0$, $x=2$, $y=1$ and $z=3$ **(15 Marks)**

85. If g_{ij} and γ_{ij} are tow metric tensors and defined at a point and $\left\{ \begin{matrix} l \\ ij \end{matrix} \right\}$ and $\left| \begin{matrix} l \\ ij \end{matrix} \right.$ are the corresponding Christoffel symbols of the second kind, then prove that $\left\{ \begin{matrix} l \\ ij \end{matrix} \right\} - \left| \begin{matrix} l \\ ij \end{matrix} \right.$ is a mixed tensor of the type A^l_{ij} **(15 Marks)**

86. Establish the formula $E=mc^2$ the symbols have their usual meaning. **(15 Marks)**

1999

87. If $\bar{a},\bar{b},\bar{c}$ are the position vectors of A,B,C prove that $\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}$ is vector perpendicular to the plane ABC **(20 Marks)**

88. If $\bar{f}=\nabla\left(x^3+y^3+z^3-3xyz\right)$ find $\nabla\times\bar{F}$. **(20 Marks)**

89. Evaluate $\int_c \left(e^{-x}\sin ydx+e^{-x}\cos ydy\right)$ (by Green's theorem), where C is the rectangle whose vertices are (0,0), $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$ **(20 Marks)**

1998

90. If r_1 and r_2 are the vectors joining the fixed points A(x_1,y_1,z_1) and $B(x_2,y_2,z_2)$ respectively to a variable point $P(x,y,z)$ then the values of grad $(r_1.r_2)$ and curl$(r_1\times r_2)$. **(20 Marks)**

91. Show that $(a\times b)\times c=a\times(b\times c)$ if either $b=0$ (or any other vector is 0) or c is collinear with a or b is orthogonal to a and c (both) **(20 Marks)**

92. Prove that $\left\{ \begin{matrix} i \\ ik \end{matrix} \right\} = \frac{\partial}{\partial x_k}\left(\log\sqrt{g}\right)$. **(20 Marks)**

1997

93. Prove that if $\bar{A},\bar{B}$ and $\bar{C}$ are there given non-coplanar vectors $\bar{F}$ then any vector can be put in the form $F=\alpha\vec{B}\times\bar{C}+\beta\bar{C}\times\bar{A}+\gamma\bar{A}\times\bar{B}$ for given determine α,β,γ.**(20 Marks)**

94. Verify Gauss theorem for $\bar{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$ **(20 Marks)**

95. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij}T_{ij}$ of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti-symmetric part of T_{ij}. **(20 Marks)**

1996

96. State and prove 'Quotient law' of tensors **(20 Marks)**

97. If $x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$ show that

(i) $\vec{r}\times grad\, f(r)=0$

(ii) $div(r^n\vec{r})=(n+3)r^n$ **(20 Marks)**

98. Verify Gauss's divergence theorem for $\overline{F}=xy\hat{i}+z^2\hat{j}+2yz\hat{k}$ on the tetrahedron $x=y=z=0$, $x+y+z=1$ **(20 Marks)**

1995

99. Consider a physical entity that is specified by twenty -seven numbers A_{ijk} in given coordinate system. In the transition to another coordinates system of this kind. Let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk}-A_{ijk}$ are the components of a tensor of third order. Is A_{ijk} the component of tensor? Give reasons for your answer **(20 Marks)**

100. Let the reason V be bounded by the smooth surfaces S and Let n denote outward drawn unit normal vector at a point on S. If ϕ is harmonic in V, Show that $\int_s \frac{\partial\phi}{\partial n}ds=0$ **(20 Marks)**

101. In the vector field $u(x)$ let there exists a surface $curl v$ on which v=0. Show that, at an arbitary point of this surface $curl v$ is tangential to the surface or vanishes. **(20 Marks)**

1994

102. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$. **(20 Marks)**

103. If $\overline{F}=y\vec{i}+(x-2xz)\overline{j}-xy\hat{k}$ evaluate $\iint_s(\Delta\times\overline{F})\vec{n}ds$ Where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy plane. **(20 Marks)**

104. Prove that $\begin{Bmatrix} i \\ ik \end{Bmatrix}=\frac{\partial}{\partial x}\left(\log\sqrt{g}\right)$. **(20 Marks)**

1993

105. Prove that the angular velocity or rotation at any point is equal to one half or the *curl* of the velocity vector V. **(20 Marks)**

106. Evaluate $\iint_s \Delta\times\overline{F}\vec{n}ds$ where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\overline{F}=z\hat{i}+x\hat{j}+y\hat{k}$ **(20 Marks)**

107. Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor or rank one

(20 Marks)

1992

108. If $\vec{F}(x,y,z)=(y^2+z^2)\vec{i}+(z^2+x^2)\vec{j}+(x^2y^2)\vec{k}$ then calculate $\int_c \vec{f}\,\vec{dx}$ where C consist of

(i) The line segment from (0,0,0) to (1,1,1)

(ii) the three line segments AB,BC and CD where A,B,C and D are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)

(iii) the curve $\vec{x}+\overline{ui}+\overline{u^2j}+\overline{u^2k},u$ from 0 to 1. **(20 Marks)**

109. If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

(i) $div\left\{x\times\left(\vec{a}\times\vec{x}\right)\right\}=-2\vec{x}\vec{a}$

(ii) $div\left\{x\times\left(\vec{a}\times\vec{x}\right)\times\left(\vec{b}\times\vec{x}\right)\right\}=-2\overline{a}\left(\vec{b}\times\vec{x}\right)-2b\left(\vec{a}\times\vec{x}\right)$ **(20 Marks)**

110. Obtain the formula $div\vec{A}=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{-1}}\left\{\left(\frac{g}{g_{ij}}\right)^{1/2}A(i)\right\}$ where $A(i)$ are physical components of $\vec{A}$ and use it to derive expression of $div\vec{A}$ in cylindrical polar coordinates

(20 Marks)