VECTOR ANALYSIS

Previous year Questions from 1992 To 2017

Syllabus

Scalar and vector fields, differentiation of vector field of a scalar variable; Gradient, divergence and curl in cartesian and cylindrical coordinates; Higher order derivatives; Vector identities and vector equations. Application to geometry: Curves in space, Curvature and torsion; Serret-Frenet’s formulae. Gauss and Stokes’ theorems, Green’s identities.

** Note: Syllabus was revised in 1990’s and 2001 & 2008 **
2017

1. Suppose $U$ and $W$ are distinct four dimensional subspaces of a vector space $V$, where $\dim V = 6$. Find the possible dimensions of subspace $U \cap W$. (10 Marks)

2. Evaluate the integral: $\int_S \vec{F} \cdot d\vec{s}$ where $\vec{F} = 3xy^2\hat{i} + (yx^2 - y^3)\hat{j} + 3xz^2\hat{k}$ and $S$ is a surface of the cylinder $y^2 + z^2 \leq 4$, $-3 \leq x \leq 3$, using divergence theorem. (9 Marks)

3. Using Green’s theorem, evaluate the $\int_C F(\vec{r}).d\vec{r}$ counterclockwise where $F(\vec{r}) = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$ and $d\vec{r} = dx\hat{i} + dy\hat{j}$ and the curve $C$ is the boundary of the region $R = \{(x, y)\mid 1 \leq y \leq 2 - x^2\}$. (8 Marks)

2016

4. Prove that the vector $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle find the length of the medians of the triangle (10 Marks)

5. Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r}$ and $f(1)=0$ (10 Marks)

6. Prove that $\oint_C f d\vec{r} = \iint_S d\vec{S} \times \nabla f$ (10 Marks)

7. For the of cardioid $r = a(1 + \cos \theta)$ show that the square of the radius of curvature at any point $(r, \theta)$ is proportion to $r$. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. (15 Marks)

2015

8. Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $x^2 + y^2 - 3$ at $(2, -1, 2)$ (10 Marks)

9. A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. (12 Marks)

10. Evaluate $\int_C e^{-x}(\sin y dx + \cos y dy)$, Where $C$ is the rectangle with vertices $(0, 0), (\pi, 0), (\pi, \frac{\pi}{2}), (0, \frac{\pi}{2})$ (12 Marks)

2014

11. Find the curvature vector at any point of the curve $\vec{r}(t) = t \cos t \hat{i} + t \sin t \hat{j}$, $0 \leq t \leq 2\pi$. Give its magnitude also. (10 Marks)
12. Evaluate by Stoke’s theorem $\int_{\Gamma} (y \, dx + z \, dy + x \, dz)$, where $\Gamma$ is the curve given by $x^2+y^2+z^2-2ax-2ay=0$, $x+y=2a$ starting from $(2a,0,0)$ and then going below the $z$-plane. (20 Marks)

2013

13. Show the curve $\vec{x}(t) = \hat{i} + \left( \frac{1+t}{t} \right) \hat{j} + \left( 1-\frac{t^2}{t} \right) \hat{k}$ lies in a plane. (10 Marks)

14. Calculate $\nabla^2 (r^n)$ and find its expression in terms of $r$ and $n$, $r$ being the distance of any point $(x,y,z)$ from the origin, $n$ being a constant and $\nabla^2$ being the Laplace operator. (10 Marks)

15. A curve in space is defined by the vector equation $\vec{r} = t^2 \hat{i} + 2t \hat{j} - t^3 \hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. (10 Marks)

16. By using Divergence Theorem of Gauss, evaluate the surface integral

$$\iint (a^2x^2 + b^2y^2 + c^2z^2)^{\frac{1}{2}} \, dS$$

where $S$ is the surface of the ellipsoid $ax^2+by^2+cz^2=1$, $a,b$ and $c$ being all positive constants. (15 Marks)

17. Use Stroke’s theorem to evaluate the line integral $\int_C (-y^2 \, dx + x^2 \, dy - z^3 \, dz)$, where $C$ is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. (15 Marks)

2012

18. If $\vec{A} = x^2yz \hat{i} - 2xz^2 \hat{j} + xz^2 \hat{k}$, $\vec{B} = 2z \hat{i} + y \hat{j} - x^2 \hat{k}$, find the value of

$$\frac{\partial^2}{\partial x \partial y} (\vec{A} + \vec{B})$$

at $(1,0,-2)$. (12 Marks)

19. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t$, $y = t^2$, $z = \frac{2}{3} t^3$. Show that the curvature and torsion are equal for this curve. (20 Marks)

20. Verify Green’s theorem in the plane for

$$\iint_C (xy + y^2 \, dx + x^2 \, dy)$$

where $C$ is the closed curve of the region bounded by $y = x$ and $y = x^2$. (20 Marks)

21. If $\vec{F} = y \hat{i} + (x - 2xz) \hat{j} - xy \hat{k}$, evaluate

$$\iiint_S (\nabla \times \vec{F}) \, d\vec{s}$$

where $S$ is the surface of the sphere $x^2+y^2+z^2=a^2$ above the $xy$-plane. (20 Marks)
22. For two vectors \( \vec{a} \) and \( \vec{b} \) give respectively by
\[ \vec{a} = 5t^2 \hat{i} + t \hat{j} - t^3 \hat{k} \]
and
\[ \vec{b} = \sin 5t \hat{i} - \cos t \hat{j} \]
determine:
(i) \( \frac{d}{dt}(\vec{a} \cdot \vec{b}) \)
(ii) \( \frac{d}{dt}(\vec{a} \times \vec{b}) \)  
\( \text{(10 Marks)} \)

23. If \( u \) and \( v \) are two scalar fields and \( \vec{f} \) is a vector field, such that \( u \vec{f} = \nabla v \), find the value of
\( \vec{f} \cdot \nabla \vec{f} \)  
\( \text{(10 Marks)} \)

24. Examine whether the vectors \( \nabla u, \nabla v, \nabla w \) are coplanar, where \( u, v \) and \( w \) are the scalar functions defined by:
\[ u = x + y + z, \]
\[ v = x^2 + y^2 + z^2 \]
and \( w = yz + zx + xy \)  
\( \text{(15 Marks)} \)

25. If \( \vec{u} = 4y \hat{i} + x \hat{j} + 2z \hat{k} \) calculate double integral
\[ \iint \nabla \times \vec{u} \, dS \]
over the hemisphere given by \( x^2 + y^2 + z^2 = a^2, \) \( z \geq 0 \)  
\( \text{(15 Marks)} \)

26. If \( \vec{r} \) be the position vector of a point, find the value(s) of \( n \) for which the vector \( \vec{r} \) is
(i) irrotational, (ii) solenoidal  
\( \text{(15 Marks)} \)

27. Verify Gauss’ Divergence Theorem for the vector \( \vec{v} = x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k} \) taken over the cube \( 0 \leq x, y, z \leq 1 \).  
\( \text{(15 Marks)} \)

2010

28. Find the directional derivative of \( f(x,y) = x^2 y^3 + xy \) at the point \( (2,1) \) in the direction of a unit vector which makes an angle or \( \frac{\pi}{3} \) with the \( x \)-axis.  
\( \text{(12 Marks)} \)

29. Show that the vector field defined by the vector function
\[ \vec{v} = xyz \left( yz \hat{i} + xy \hat{j} + xy \hat{k} \right) \]
is conservative.  
\( \text{(12 Marks)} \)

30. Prove that \( \text{div}(f \vec{V}) = f \left( \text{div} \vec{V} \right) + \left( \text{grad} \cdot f \right) \vec{V} \) where \( f \) is a scalar function.  
\( \text{(20 Marks)} \)

31. Use the divergence theorem to evaluate
\[ \iiint S \vec{V} \cdot dA \]
where \( \vec{V} = x^2 \hat{i} + y \hat{j} - xz^2 \hat{k} \) and \( S \) is the boundary of the region bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 4y \).  
\( \text{(20 Marks)} \)

32. Verify Green’s theorem for \( e^x \sin y \, dx + e^x \cos y \, dy \) the path of integration being the boundary of the square whose vertices are \((0,0), \left( \frac{\pi}{2}, 0 \right), \left( \frac{\pi}{2}, \frac{\pi}{2} \right), \text{ and } \left( 0, \frac{\pi}{2} \right) \)  
\( \text{(20 Marks)} \)

2009

33. Show that \( \text{div}(\text{grad} \phi) = n(n+1)r^{n-2} \) where \( r = \sqrt{x^2 + y^2 + z^2} \).  
\( \text{(12 Marks)} \)
34. Find the directional derivative of (i) \(4xz^3-3x^2y^2z^2\) at (2,−1,1) along z-axis
   (ii) \(-x^2yz+4xz^2\) at (1,−2,1) in the direction of \(2\hat{i} - \hat{j} - 2\hat{k}\). \((6+6=12\text{ Marks})\)

35. Find the work done in moving the particle once round the ellipse \(\frac{x^2}{25} + \frac{y^2}{16} = 1, z=0\)
   under the field of force given by \(\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}\). \((20\text{ Marks})\)

36. Using divergence theorem, evaluate \(\iint_S \vec{A} \cdot d\vec{S}\) where \(\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}\) and \(S\) is the surface of the sphere \(x^2+y^2+z^2=a^2\) \((20\text{ Marks})\)

37. Find the value of \(\iiint_S (\nabla \times \vec{f}) \cdot d\vec{S}\) taken over the upper portion of the surface \(x^2+y^2-2ax+az=0\) and the bounding curve lies in the plane \(z=0\), when
   \(\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}\) \((20\text{ Marks})\)

2008

38. Find the constants \(a\) and \(b\) so that the surface \(ax^2-byz=(a+2)x\) will be orthogonal to the surface \(4x^2y+z^3=4\) at the point \((1,-1,2)\). \((12\text{ Marks})\)

39. Show that \(\vec{F} = (2xy+z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}\) is a conservative force field. Find the scalar potential for \(\vec{F}\) and the work done in moving an object in this field \((1,-2,1)\) to \((3,1,4)\). \((12\text{ Marks})\)

40. Prove that \(\nabla^2 f(x) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}\) where \(r = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}\). Hence find \(f(x)\) such that \(\nabla^2 f(x) = 0\). \((15\text{ Marks})\)

41. Show that for the space curve \(x=t, y=t^2, z=\frac{2}{3}t^3\) the curvature and torsion are same at every point. \((15\text{ Marks})\)

42. Evaluate \(\int_c \vec{A} \cdot d\vec{r}\) along the curve \(x^2+y^2=1, z=1\) from \((0,1,1)\) to \((1,0,1)\) if
   \(\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}\). \((15\text{ Marks})\)

43. Evaluate \(\iint_S \vec{F} \cdot d\vec{s}\) where \(\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}\), \(\iint_S \vec{F} \cdot d\vec{s}\) and \(S\) is the surface of the cylinder bounded by \(x^2+y^2=4, z=0\) and \(z=3\) \((15\text{ Marks})\)
2007

44. If \( \vec{r} \) denotes the position vector of a point and if \( \hat{r} \) be the unit vector in the direction of \( \vec{r}, r = |\vec{r}| \) determined grad \((r^{-1})\) in terms of \( \hat{r} \) and \( r \). (12 Marks)

45. Find the curvature and torsion at any point of the curve \( x = a \cos 2t, y = a \sin 2t, z = 2a \sin t \). (12 Marks)

46. For any constant vector, show that the vector \( \vec{a} \) represented by \( \text{curl} (\vec{a} \times \vec{r}) \) is always parallel to the vector \( \vec{a}, \vec{r} \) being the position vector of a point \((x,y,z)\) measured from the origin. (15 Marks)

47. If \( \vec{r} = xi + yj + zk \) find the value(s) of \( n \) in order that \( n \vec{r} \) may be (i) solenoidal (ii) irrotational (15 Marks)

48. Determine \( \int_C (ydx + zdz + xdz) \) by using Stoke's theorem, where \( C \) is the curve defined by \((x-a)^2 + (y-a)^2 + z^2 = 2a^2, x+y=2a\) that starts from the point \((2a,0,0)\) goes at first below the \(z\)-plane (15 Marks)

49. Find the values of constants \( a, b \) and \( c \) so that the directional derivative of the function \( f = axy^2 + byz + cz^2x^2 \) at the point \((1,2,-1)\) has maximum magnitude 64 in the direction parallel to \(z\)-axis. (12 Marks)

50. If \( A = 2i + K, B = i + j + k, C = 4i - 3j - 7K \) determine a vector \( \vec{R} \) satisfying the vector equation \( \vec{R} \times \vec{B} = \vec{C} \times \vec{B} \) & \( \vec{R} \cdot \vec{A} = 0 \) (15 Marks)

51. Prove that \( n \vec{r} \) is an irrotational vector for any value of \( n \) but is solenoidal only if \( n+3 = 0 \) (15 Marks)

52. If the unit tangent vector \( \vec{t} \) and binormal \( \vec{b} \) make angles \( \phi \) and \( \phi \) respectively with a constant unit vector \( \vec{a} \) prove that \( \frac{\sin \theta}{\sin \phi} \cdot \frac{d\theta}{d\phi} = -\frac{k}{\tau} \). (15 Marks)

53. Verify Stoke's theorem for the function \( \vec{F} = x^2 \hat{i} - xy \hat{j} \) integrated round the square in the plane \( z = 0 \) and bounded by the lines \( x = 0, y = 0, x = a \) and \( y = a, a > 0 \). (15 Marks)

2006

54. Show that the volume of the tetrahedron \( ABCD \) is \( \frac{1}{6}(|\vec{AB} \times \vec{AC}| \cdot \vec{AD}) \). Hence find the volume of the tetrahedron with vertices \((2,2,2), (2,0,0), (0,2,0)\) and \((0,0,2)\) (12 Marks)

55. Prove that the curl of a vector field is independent of the choice of coordinates (12 Marks)

2005

54. Show that the volume of the tetrahedron \( ABCD \) is \( \frac{1}{6}(|\vec{AB} \times \vec{AC}| \cdot \vec{AD}) \). Hence find the volume of the tetrahedron with vertices \((2,2,2), (2,0,0), (0,2,0)\) and \((0,0,2)\) (12 Marks)

55. Prove that the curl of a vector field is independent of the choice of coordinates (12 Marks)
56. The parametric equation of a circular helix is \( r = a \cos u \hat{i} + a \sin u \hat{j} + cu \hat{k} \) where \( c \) is a constant and \( u \) is a parameter. Find the unit tangent vector \( \hat{t} \) at the point \( u \) and the arc length measured form \( u=0 \). Also find \( \frac{d\hat{t}}{ds} \) where \( S \) is the arc length. (15 Marks)

57. Show that \( \text{curl} \left( k \times \text{grad} \left( \frac{1}{r} \right) \right) + \text{grad} \left( k \cdot \text{grad} \left( \frac{1}{r} \right) \right) = 0 \) where \( r \) is the distance from the origin and \( K \) is the unit vector in the direction \( OZ \). (15 Marks)

58. Find the curvature and the torsion of the space curve. (15 Marks)

59. Evaluate \( \iiint x^2 dy dz + x^2 y dz dx + x^2 z dx dy \) by Gauss divergence theorem, where \( S \) is the surface of the cylinder \( x^2 + y^2 = a^2 \) bounded by \( z = 0 \) and \( x = b \). (15 Marks)

2004

60. Show that if \( \overrightarrow{A} \) and \( \overrightarrow{B} \) are irrotational, then \( \overrightarrow{A} \times \overrightarrow{B} \) is solenodial. (12 Marks)

61. Prove the identity \( \nabla \cdot (A \times B) = (B \cdot \nabla)A + (A \cdot \nabla)B + B \times (\nabla \times A) + A \times (\nabla \times B) \). (15 Marks)

62. Prove the identity \( \iiint (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \iint ((\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}) dS \) where \( V \) is the volume bounded by the closed surface \( S \). (15 Marks)

63. Verify Stoke's theorem for \( \hat{f} = (2x - y) \hat{i} - yz^2 \hat{j} - z \hat{k} \) where \( S \) is the upper half surface of the sphere \( x^2 + y^2 + z^2 = 1 \) and \( C \) is its boundary. (15 Marks)

2003

65. Show that if \( a', b' \) and \( c' \) are the reciprocals of the non-coplanar vectors \( a, b \) and \( c \), then any vector \( r \) may be expressed as \( r = (r a') a + (r b') b + (r c') c \). (12 Marks)

66. Prove that the divergence of a vector field is invariant w.r. to co-ordinate transformations. (12 Marks)

67. Let the position vector of a particle moving on a plane curve be \( r(t) \), where \( t \) is the time. Find the components of its acceleration along the radial and transverse directions. (15 Marks)

68. Prove the identity \( \nabla A^2 = 2 (A \cdot \nabla) A + 2 A \times (\nabla \times A) \) where \( \nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \). (15 Marks)
69. Find the radii of curvature and torsion at a point of intersection of the surface
\[ x^2 - y^2 = c^2, \ y = x \tanh \left( \frac{z}{c} \right). \] (15 Marks)

70. Evaluate \( \iint_S \text{curl} \mathbf{A} \, ds \) Where S is the open surface \( x^2 + y^2 - 4x + 4z = 0, \ z \geq 0 \) and
\[ A = \left( y^2 + z^2 - x^2 \right) \mathbf{i} + \left( 2z^2 + x^2 - y^2 \right) \mathbf{j} + \left( x^2 + y^2 - 3z^2 \right) \mathbf{k}. \] (15 Marks)

2002

71. Let \( \mathbf{R} \) be the unit vector along the vector \( \mathbf{r}(t) \). Show that \( \mathbf{R} \times \frac{d\mathbf{R}}{dt} = \frac{\mathbf{r}}{r^2} \times \frac{d\mathbf{r}}{dt} \) where \( r = |\mathbf{r}| \) (12 Marks)

72. Find the curvature \( k \) for the space curve \( x = \cos \theta, \ y = \sin \theta, \ z = a \theta \tan \alpha \) (15 Marks)

73. Show that \( (\text{curl} \mathbf{v}) = \nabla (\text{div} \mathbf{v}) - \nabla^2 \mathbf{v} \) (15 Marks)

74. Let \( D \) be a closed and bounded region having boundary \( S \). Further, let \( f \) is a scalar function having second partial derivatives defined on it. Show that
\[ \iint_S (\nabla f) \cdot \mathbf{n} \, ds = \iiint_D \left[ |\nabla f|^2 + f \nabla^2 f \right] \, dv \] Hence \( \iint_S (\nabla f) \cdot \mathbf{n} \, ds \) or otherwise evaluate for \( f = 2x + y + 2z \) over \( s = x^2 + y^2 + z^2 = 4 \) (15 Marks)

75. Find the values of constants \( a, b \) and \( c \) such that the maximum value of directional derivative of \( f = axy^2 + byz + cx^2z^2 \) at \( (1, -1, 1) \) is in the direction parallel to y-axis and has magnitude 6. (15 Marks)

2001

76. Find the length of the arc of the twisted curve \( r = (3t, 3t^2, 2t^3) \) from the point \( t=0 \) to the point \( t=1 \). Find also the unit tangent \( t \), unit normal \( n \) and the unit binormal \( b \) at \( t=1 \) (12 Marks)

77. Show that \( \text{curl} \frac{a \times r}{r^3} = -\frac{a}{r^3} + \frac{3r}{r^3} (a \cdot r) \) where \( a \) is a constant vector. (12 Marks)

78. Find the directional derivative of \( f = x^2yz^3 \) along \( x = e^{-t}, y = 1 + 2\sin t, z = t - \cos t \) at \( t=0 \) (15 Marks)

79. Show that the vector field defined by \( F = 2xyz^2 i + x^2z^3 j + 3x^3yz^2 k \) is irrotational. Find also the scalar \( u \) such that \( F = \nabla u \) (15 Marks)

80. Verify Gauss’ divergence theorem of \( A = (4x, -2y^2, z^2) \) taken over the region bounded by \( x^2 + y^2 = 4, \ z = 0 \) and \( z = 3 \) (15 Marks)

2000

81. In what direction from the point \( (-1, 1, 1) \) is the directional derivative \( f = x^2yz^3 \) a maximum? Compute its magnitude (12 Marks)
82. (i) Show that the covariant derivatives of the fundamental metric tensors $g_{ij}$, $\delta_{ij}$, Vanish
(ii) Show that simultaneity is relative in special relativity theory. (6+6=12 Marks)

83. Show that
(i) $(A+B) \cdot (B+C) \cdot (C+A) = 2A \cdot B \cdot C$ 
(ii) $\nabla \times (A \times B) = (B \cdot \nabla) A - (A \cdot \nabla) B + A (\nabla \cdot B)$ (7+8=15 Marks)

84. Evaluate
$$\int \int F \cdot N ds$$ Where $F=2xyi + yz^2j + xzk$ and $S$ is the surface of the parallelepiped bounded by $x=0, y=0, z=0, x=2, y=1$ and $z=3$ (15 Marks)

85. If $g_{ij}$ and $\gamma_{ij}$ are two metric tensors and defined at a point and $l_{ij}$ and $\Lambda_{ij}$ are the corresponding Christoffel symbols of the second kind, then prove that $l_{ij} - \Lambda_{ij}$ is a mixed tensor of the type $A_{ij}$ (15 Marks)

86. Establish the formula $E=mc^2$ the symbols have their usual meaning. (15 Marks)

87. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of $A, B, C$ prove that $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is vector perpendicular to the plane $ABC$ (20 Marks)

88. If $\vec{F} = \nabla \left( x^3 + y^3 + z^3 - 3xyz \right)$ find $\nabla \times \vec{F}$. (20 Marks)

89. Evaluate $\int_C \left( e^{-x} \sin y dx + e^{-y} \cos y dy \right)$ (by Green’s theorem), where $C$ is the rectangle whose vertices are $(0,0), (\pi,0), \left( \pi, \frac{\pi}{2} \right)$ and $\left( 0, \frac{\pi}{2} \right)$ (20 Marks)

90. If $r_1$ and $r_2$ are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point $P(x, y, z)$ then the values of grad $(r_1, r_2)$ and curl $(r_1, r_2)$. (20 Marks)

91. Show that $(a \times b) \times c = a \times (b \times c)$ if either $b=0$ (or any other vector is 0) or $c$ is collinear with $a$ or $b$ is orthogonal to $a$ and $c$ (both) (20 Marks)

92. Prove that $\left\{ \frac{i}{ik} \right\} = \frac{\partial}{\partial x_k} \left( \log \sqrt{g} \right)$. (20 Marks)

93. Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are there given non-coplanar vectors $\vec{F}$ then any vector can be put in the form $F = \alpha \vec{B} \times \vec{C} + \beta \vec{C} \times \vec{A} + \gamma \vec{A} \times \vec{B}$ for given determine $\alpha, \beta, \gamma$ (20 Marks)

94. Verify Gauss theorem for $\vec{F} = 4x \hat{i} - 2y^2 \hat{j} + z^2 \hat{k}$ taken over the region bounded by $x^2+y^2=4, z=0$ and $z=3$ (20 Marks)
95. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij} T_{ij}$ of a tensor $T_{ij}$ with a symmetric tensor $S_{ij}$ is independent of the anti-symmetric part of $T_{ij}$. (20 Marks)

1996

96. State and prove ‘Quotient law’ of tensors (20 Marks)

97. If $\hat{x} i + y \hat{j} + z \hat{k}$ and $r = |\vec{r}|$ show that
   (i) $\vec{r} \times \nabla f(r) = 0$
   (ii) $\nabla \cdot (r^n \vec{r}) = (n+3)r^n$ (20 Marks)

98. Verify Gauss’s divergence theorem for $\vec{F} = x \hat{i} + y \hat{j} + z^2 \hat{k}$ on the tetrahedron $x=y=z=0, x+y+z=1$ (20 Marks)

1995

99. Consider a physical entity that is specified by twenty-seven numbers $A_{ijk}$ in a given coordinate system. In the transition to another coordinate system of this kind, let $A_{ijk}, B_{ijk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk} - A_{ijk}$ are the components of a tensor of third order. Is $A_{ijk}$ the component of tensor? Give reasons for your answer (20 Marks)

100. Let the reason $V$ be bounded by the smooth surfaces $S$ and let $n$ denote outward drawn unit normal vector at a point on $S$. If $\phi$ is harmonic in $V$, show that $\int \frac{\partial \phi}{\partial n} ds = 0$ (20 Marks)

101. In the vector field $u(x)$ let there exists a surface curl $v$ on which $v = 0$. Show that, at an arbitrary point of this surface curl $v$ is tangential to the surface or vanishes. (20 Marks)

1994

102. Show that $n^a j^b$ is an irrotational vector for any value of $n$, but is solenoidal only if $n = -3$. (20 Marks)

103. If $\vec{F} = y \hat{i} + (x - 2xz) \hat{j} - xy \hat{k}$ evaluate $\iint_s (\Delta \times \vec{F}) \cdot n ds$ Where $S$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the $xy$ plane. (20 Marks)

104. Prove that $\left\{ i \atop ik \right\} = \frac{\partial}{\partial x} \left( \log \sqrt{g} \right)$. (20 Marks)

1993

105. Prove that the angular velocity or rotation at any point is equal to one half or the curl of the velocity vector $V$. (20 Marks)

106. Evaluate $\iint_S \Delta \times \vec{F} \cdot n ds$ where $S$ is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z \hat{i} + x \hat{j} + y \hat{k}$ (20 Marks)
107. Show that \( \frac{\partial A_p}{\partial x^q} \) is not a tensor even though \( A_p \) is a covariant tensor or rank one.

(20 Marks)

1992

108. If \( \mathbf{F}(x, y, z) = (y^2 + z^2)\mathbf{i} + (z^2 + x^2)\mathbf{j} + (x^2 y^2)\mathbf{k} \) then calculate \( \int_C \mathbf{F} \cdot d\mathbf{x} \) where \( C \) consist of

(i) The line segment from (0,0,0) to (1,1,1)
(ii) the three line segments AB, BC and CD where \( A, B, C \) and \( D \) are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)
(iii) the curve \( x = u, u^2 + 2uj + u^2k, u \) from 0 to 1.

(20 Marks)

109. If \( \mathbf{a} \) and \( \mathbf{b} \) are constant vectors, show that

(i) \( \text{div}\{x \times (\mathbf{a} \times \mathbf{x})\} = -2x\mathbf{a} \)

(ii) \( \text{div}\{x \times (\mathbf{a} \times \mathbf{x}) \times (\mathbf{b} \times \mathbf{x})\} = -2\mathbf{a}((\mathbf{b} \times \mathbf{x}) \cdot \mathbf{x}) - 2\mathbf{b}((\mathbf{a} \times \mathbf{x}) \cdot \mathbf{x}) \)

(20 Marks)

110. Obtain the formula \( \text{div}\mathbf{A} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left( \frac{g_i}{g} \right)^{1/2} A(i) \) where \( A(i) \) are physical components of \( \mathbf{A} \) and use it to derive expression of \( \text{div}\mathbf{A} \) in cylindrical polar coordinates

(20 Marks)