(1) Number of subgroups of $Z_n = \tau(n)$

$=$ number of divisors of n

If $n = p_1^{\alpha_1} \times p_2^{\alpha_2} \times \cdots \times p_r^{\alpha_r}$

where $p_1, p_2 \ldots p_r$ are prime numbers

then $\tau(n) = (\alpha_1+1)(\alpha_2+1) \cdots (\alpha_r+1)$

Ex: find number of subgroups of Z_{20}

sol: $n = 20 = 2^2 \times 5^1$

number of subgroups of $Z_{20} = (2+1)(1+1) = 6$

(2) Number of generators of a finite cyclic group $(G, \cdot)$ of order n $= \varphi(n)$

If $n = p_1^{\alpha_1} \times p_2^{\alpha_2} \times \cdots p_r^{\alpha_r}$

then $\varphi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2}) \cdots (1-\frac{1}{p_r})$

Prob: Find number of generators of cyclic group (1) of order 15

(2) $(Z_{12}, +_{12})$

(1) $n = 15$, number of generators $= \varphi(n) = \varphi(15)$

$= \varphi(3^1 \times 5^1) = 15(1-\frac{1}{3})(1-\frac{1}{5})$

$= 15 \times \frac{2}{3} \times \frac{4}{5} = 8$

number of generators $= \varphi(12)$

$= \varphi(2^2 \times 3^1)$

$= 6\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$

$= 6 \times \frac{1}{2} \times \frac{2}{3} = 2.$

(3) Number of subrings of $(\mathbb{Z}_n, +_n, \times_n)$

$=$ Number of ideals of $(\mathbb{Z}_n, +_n, \times_n)$

$= \tau(n).$

Prob: Find number of subrings of $(\mathbb{Z}_{10}, +_{10}, \times_{10})$. Also find number of ideals.

Sol:- number of subrings $=$ number of ideals $= \tau(10) = \tau(2^1 \times 5^1)$

$= (1+1)(1+1) = 4$

(4) Prob Find number of ideals of $\mathbb{Z}_5 \times \mathbb{Z}_{10}$

Sol: number of ideals $= \tau(5) \times \tau(10)$

$= \tau(5^1) \times \tau(2^1 \times 5^1)$

$= (1+1) \times (1+1) \times (1+1)$

$= 8$

(5) Prob Find number of ideals of $Q \times R \times \mathbb{Z}_3$

Sol: Every field has two ideals.

since $Q, R, \mathbb{Z}_3$ are fields

number of ideals of $Q \times R \times \mathbb{Z}_3 = 2 \times 2 \times 2 = 8.$

(6) Number of maximal ideals in $(Z_n, +_n, \times_n)$ (3)

Number of prime ideals = Number of prime divisors of n

Qn: Find number of maximal ideals and number of prime ideals of $(Z_{14}, +_{14}, \times_{14})$

sol: $n = 14 \Rightarrow$ divisors of 14 = 1, 2, 7, 14

Prime divisors of 14 = 2, 7.

∴ Number of prime ideals of Z_{14}

= Number of Maximal ideals of Z_{14}

= 2.

(7) Number of units of $(Z_n, +_n, \times_n)$

$= \varphi(n)$

Qn: Find number of units of $(Z_{24}, +_{24}, \times_{24})$

sol: Number of units of $Z_{24} = \varphi(24)$

$= \varphi(2^3 \times 3^1)$

$= 24(1-\frac{1}{2})(1-\frac{1}{3}) = 24 \times \frac{1}{2} \times \frac{2}{3}$

$= 8$

(8) Number of idempotent elements in $Z_n = 2^d$

d = number of prime divisors of n

(4)

Prob: Find numbers of idempotent elements of $(Z_6, +_6, \times_6)$.

Sol: $n = 6$. Prime divisors of 6 are 2, 3.

Number of prime divisors of 6 $= 2 = d$

Number of idempotent elements $= 2^2 = 4$.