

# IAS



## MATHEMATICS ORDINARY DIFFERENTIAL EQUATION

Previous year Questions from **1992 To 2017**

### Syllabus

Formulation of differential equations; Equations of first order and first degree, integrating factor; Orthogonal trajectory; Equations of first order but not of first degree, Clairaut's equation, singular solution. Second and higher order linear equations with constant coefficients, complementary function, particular integral and general solution.

Second order linear equations with variable coefficients, Euler-Cauchy equation; Determination of complete solution when one solution is known using method of variation of parameters.

Laplace and Inverse Laplace transforms and their properties; Laplace transforms of elementary functions. Application to initial value problems for 2nd order linear equations with constant coefficients.

**\*\* Note: Syllabus was revised in 1990's and 2001 & 2008 \*\***



**Corporate Office:** 2<sup>nd</sup> Floor, 1-2-288/32, Indira Park 'X' Roads, Domalguda, Hyderabad-500 029.  
Ph: 040-27620440, 9912441137/38, Website: [www.analogeducation.in](http://www.analogeducation.in)

**Branches:** **New Delhi:** Ph:8800270440, 8800283132 **Bangalore:** Ph: 9912441138, 9491159900 **Guntur:** Ph:9963356789 **Vishakapatnam:** Ph: 08912546686

# 2017

1. Solve the differential equation  $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$  (9 marks)

2. Solve the following differential equation using method of variation of parameters:

$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2 \quad (8 \text{ marks})$$

3. Solve the following initial value problem using laplace transform

$$\frac{d^2 y}{dx^2} + 9y = r(x) \text{ Where } r(x) = \begin{cases} y(0) = 0 & y'(0) = 4 \\ 8 \sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases} \quad (17 \text{ marks})$$

4. Solve the following initial value differential equations:

$$20y'' + 4y' + y = 0, y(0) = 3.2 \text{ and } y'(0) = 0 \quad (7 \text{ marks})$$

5. Consider the differential equation  $xy p^2 - (x^2 + y^2 - 1)p + xy = 0$  where  $p = \frac{dy}{dx}$ . Substituting

$$u = x^2 \text{ and } v = y^2 \text{ reduce the equation to clairaut's form in terms of } u \text{ and } v \text{ } p' = \frac{dv}{du}.$$

Hence, or otherwise solve the equation (10 marks)

6. Solve the following simultaneous linear differential equations:  $(D+1)y = z + e^x$  and

$$(D+1)z = y + e^x \text{ where } y \text{ and } z \text{ are functions of independent variable } x \text{ and } D = \frac{d}{dx}$$

(8 marks)

7. Find the differential equation representing all the circles in the  $xy$  plane (10 marks)

8. Suppose that the streamlines of the fluid flow are given by a family of curves  $xy = c$ . Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. (10 marks)

# 2016

9. Find a particular integral of  $\frac{d^2 y}{dx^2} + y = e^{x/2} \sin \frac{x\sqrt{3}}{2}$  (10 marks)

10. Show that the family of parabolas  $y^2 = 4cx + 4c^2$  is self orthogonal. (10 marks)

11. Solve  $\{y(1 - x \tan x) + x^2 \cos x\} dx - x dy = 0$  (10 marks)

12. Using the method of variation of parameter solve the differential equation

$$(D^2 + 2D + 1)y = e^{-x} \log(x), \left[ D = \frac{d}{dx} \right] \quad (15 \text{ marks})$$

13. Find the general solution of the equation  $x^2 \frac{d^3 y}{dx^3} - 4x \frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} = 4$  (10 marks)
14. Using Laplace transformation solves the following:  $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$  (10 marks)

## 2015

15. Solve the differential equation:  $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$  (10 marks)
16. Solve the differential equation:  $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$  (10 marks)
17. Find the constant  $a$  so that  $(x+y)^a$  is the integrating factor of  $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$  and hence solve the differential equation (12 marks)
18. (i) Obtaining Laplace Inverse transform of  $\left\{ \ln \left( 1 + \frac{1}{s^2} \right) + \frac{s}{s^2 + 25} e^{-5s} \right\}$   
(ii) Using Laplace transform, Solve  $y'' + y = t, y(0) = 1, y'(0) = -2$  (12 marks)
19. Solve the differential equation  $x = py - p^2$  where  $p = \frac{dy}{dx}$  (13 marks)
20. Solve  $x^4 \frac{d^4 y}{dx^4} + 6x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \cos(\log_e x)$  (13 marks)

## 2014

21. Justify that a differential equation of the form:  $[y + xf(x^2 + y^2)]dx + [yf(x^2 + y^2) - x]dy = 0$  Where  $f(x^2 + y^2)$  is an arbitrary function of  $(x^2 + y^2)$ , is not an exact differential equation and  $\frac{1}{x^2 + y^2}$  is an integrating factor for it. Hence solve this differential equation for  $f(x^2 + y^2) = (x^2 + y^2)^2$  (10 marks)
22. Find the curve for which the part of tangent cut-off by the axes is bisected at the point of tangency (10 marks)
23. solve by the method of variation of parameters:  $\frac{dy}{dx} - 5y = \sin x$  (10 marks)
24. Solve the differential equation:  $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log_e x)$  (20 marks)
25. Solve the following differential equation:  $x \frac{d^2 y}{dx^2} - 2(x+1) \frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$ , when  $e^x$  is a solution to its corresponding homogeneous differential equation. (15 marks)
26. Find the sufficient condition for the differential equation  $M(x,y)dx + N(x,y)dy = 0$ , to have an integrating factor as a function of  $(x+y)$ . What will be the integrating factor in that case? Hence find the integrating factor for the differential equation of

$x^2+xy)dx+(y^2+xy)dy=0$  and solve it. **(15 marks)**

27. Solve the initial value problem  $\frac{d^2y}{dt^2} + y = 8e^{-2t} \sin t, y(0), y'(0) = 0$  By using Laplace transform. **(20 marks)**

## 2013

28. If  $y$  is a function of  $x$ , such that the differential coefficient  $\frac{dy}{dx}$  is equal to  $\cos(x+y) + \sin(x+y)$ . Find out a relation between  $x$  and  $y$ , which is free from any derivative / differential. **(10 marks)**
29. Obtain the equation of the orthogonal trajectory of the family of curves represented by  $r^n = a \sin n\theta$ ,  $(r, \theta)$  being the plane polar coordinates. **(10 marks)**
30. Solve the differential equation  $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$  **(15 marks)**
31. Using the method of variation of parameters, solve the differential equation

$$\frac{d^2y}{dx^2} + a^2y = \sec ax \quad \textbf{(15 marks)}$$

32. Find the general solution of the equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \ln x \sin(\ln x)$  **(15 marks)**

33. By using Laplace transform method, solve the differential equation

$(D^2 + n^2)x = a \sin(nt + \alpha)$ ,  $D^2 = \frac{d^2}{dt^2}$  subject to the initial conditions  $x=0$  and  $\frac{dx}{dt} = 0$ , at  $t=0$ , in which  $a, n$  and  $\alpha$  are constants. **(15 marks)**

## 2012

34. Solve  $\frac{dy}{dx} = \frac{2xy^2 e^{(x/y)^2}}{y^2(1 + e^{(x/y)^2}) + 2x^2 e^{(x/y)^2}}$  **(12 marks)**

35. Find the orthogonal trajectory of the family of curves  $x^2 + y^2 = ax$  **(12 marks)**

36. Using Laplace transforms, solve the initial value problem  $y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$  **(10 marks)**

37. Show that the differential equation  $(2xy \log y) dx + (x^2 + y^2 \sqrt{y^2 + 1}) dy = 0$  is not exact.

Find an integrating factor and hence, the solution of the equation **(20 marks)**

38. Find the general solution of the equation  $y''' - y'' = 12x^2 + 6x$  **(20 marks)**

39. Solve the ordinary differential equation  $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$  **(20 marks)**

## 2011

40. Obtain the solution of the ordinary differential equation  $\frac{dy}{dx} = (4x + y + 1)^2$ , if  $y(0) = 1$  **(10 marks)**

41. Determine the orthogonal trajectory of a family of curves represented by the polar equation  $r = a(1 - \cos \theta)$ ,  $(r, \theta)$  being the plane coordinates of any point.

**(10 marks)**

42. Obtain Caliraut's form of the differential equation  $\left(x \frac{dy}{dx} - y\right)\left(y \frac{dy}{dx} + x\right) = a^2 \frac{dy}{dx}$ . Also find its general solution **(15 marks)**
43. Obtain the general solution of the second order ordinary differential equation  $y'' - 2y' + 2y = x + e^x \cos x$ , where dashes denote derivatives w.r.t.  $x$  **(15 marks)**
44. Using the method of variation of parameters, solve the second order differential equation  $\frac{d^2y}{dx^2} + 4y = \tan 2x$  **(15 marks)**
45. Use Laplace transform method to solve the following initial value problem:  
 $\frac{d^2x}{dt^2} - 2 \frac{dx}{dt} + x = e^t, x(0) = 2$  and  $\left. \frac{dy}{dx} \right|_{t=0} = -1$  **(15 marks)**

## 2010

46. Consider the differential equation  $y' = \alpha x, x > 0$  where  $\alpha$  is a constant. Show that  
 (i) If  $\phi(x)$  is any solution and  $\psi(x) = \phi(x)e^{-\alpha x}$ , then  $\psi(x)$  is a constant;  
 (ii) If  $\alpha < 0$ , then every solution tends to zero as  $x \rightarrow \infty$  **(12 marks)**
47. Show that the differential equation  $(3y^2 - x) + 2y(y^2 - 3)y' = 0$  admits an integrating factor which is a function of  $(x + y^2)$ . Hence solve the equation. **(12 marks)**
48. Verify that  $\frac{1}{2}(Mx + Ny)d[\log_e(xy)] + \frac{1}{2}(Mx - Ny)d[\log_e(x/y)] = Mdx + Ndy$ . Hence show that  
 (i) If the differential equation  $Mdx + Ndy = 0$  is homogeneous, then  $(Mx + Ny)$  is an integrating factor unless  $Mx + Ny = 0$ ;  
 (ii) If the differential equation  $Mdx + Ndy = 0$  is not exact but is of the form  $f_1(xy)ydx + f_2(xy)xdy = 0$  then  $(Mx - Ny)^{-1}$  is an integrating factor unless  $Mx + Ny = 0$ ;  
**(20 marks)**
49. Use the method of undermined coefficients to find the particular solutions of  $y'' + y = \sin x + (1 + x^2)e^x$  and hence find its general solution. **(20 marks)**

## 2009

50. Find the Wronskian of the set of functions:  $\{3x^3, |3x^3|\}$  on the interval  $[-1, 1]$  and determine whether the set is linearly dependent on  $[-1, 1]$  **(12 marks)**
51. Find the differential equation of the family of circles in the  $xy$ -plane passing through  $(-1, 1)$  and  $(1, 1)$  **(20 marks)**
52. Find the inverse Laplace transform of  $F(s) = \ln\left(\frac{s+1}{s+5}\right)$  **(20 marks)**
53. Solve:  $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1$  **(20 marks)**

## 2008

54. Solve the differential equation  $yx + (x + x^3y^2)dy = 0$  (12 marks)
55. Use the method of variation of parameters to find the general solution of  $x^2y'' - 4xy' + 6y = -x^4 \sin x$  (12 marks)
56. Using Laplace transform, solve the initial value problem  $y'' - 3y' + 2y = 4t + e^{3t}$ ,  $y(0) = 1$ ,  $y'(0) = -1$  (15 marks)
57. Solve the differential equation  $x^3y'' - 3x^2y' + xy = \sin(1nx) + 1$  (15 marks)
58. Solve the equation  $y - 2xp + yp^2 = 0$ , where  $p = \frac{dy}{dx}$  (15 marks)

## 2007

59. Solve the ordinary differential equation  $\cos 3x \frac{dy}{dx} - 3y \sin 3x = \frac{1}{2} \sin 6x + \sin^2 3x$ ,  $0 < x < \frac{\pi}{2}$  (12 marks)
60. Find the solution of the equation  $\frac{dy}{y} + xy^2 dx = -4x dx$  (12 marks)
61. Determine the general and singular solutions of the equation  $y = x \frac{dy}{dx} + a \frac{dy}{dx} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}}$ ,  $a$  being a constant. (15 marks)
62. Obtain the general solution of  $[D^3 - 6D^2 + 12D - 8]y = 12 \left[ e^{2x} + \frac{9}{4} e^{-x} \right]$ , where  $D = \frac{dy}{dx}$  (15 marks)
63. Solve the equation  $2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^3$  (15 marks)
64. Use the method of variation of parameters to find the general solution of the equation  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = 2e^x$  (15 marks)

## 2006

65. Find the family of curves whose tangents form an angle  $\frac{\pi}{4}$  with the hyperbolas  $xy = c$ ,  $c > 0$  (12 marks)
66. Solve the differential equation  $\left( xy^2 + e^{-\frac{1}{x^3}} \right) dx - x^2 y dy = 0$  (12 marks)
67. Solve :  $(1 + y^2) + (x - e^{-\tan^{-1}y}) \frac{dy}{dx} = 0$  (15 marks)

68. Solve the equation  $x^2 p^2 + py(2x + y) + y^2 = 0$  using the substitution  $y = u$  and  $xy = v$  and find its singular solution, where  $p = \frac{dy}{dx}$  (15 marks)
69. Solve the differential equation  $x^2 \frac{d^3 y}{dx^3} + 2x \frac{d^2 y}{dx^2} + 2 \frac{y}{x} = 10 \left( 1 + \frac{1}{x^2} \right)$  (15 marks)
70. Solve the differential equation  $(D^2 - 2D + 2)y = e^x \tan x$ ,  $D = \frac{dy}{dx}$  by the method of variation of parameters. (15 marks)

## 2005

71. Find the orthogonal trajectory of the family of co-axial circles  $x^2 + y^2 + 2gx + c = 0$ , where  $g$  is the parameters. (12 marks)
72. Solve :  $xy \frac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2 y^2 - 1)}$  (12 marks)
73. Solve the differential equation:  $\left[ (x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1) \right] y = \frac{1}{(x+1)}$  (15 marks)
74. Solve the differential equation:  $(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$  where  $p = \frac{dy}{dx}$ , by reducing it to Clairaut's form by using suitable substitution. (15 marks)
75. Solve the differential equation  $(\sin x - x \cos x)y'' - x \sin x y' + y \sin x = 0$  given that  $y = \sin x$  is a solution of this equation. (15 marks)
76. Solve the differential equation  $x^2 y'' - 2xy' + 2y = x \log x$ ,  $x > 0$  by variation of parameters (15 marks)

## 2004

77. Find the solution of the following differential equation  $\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x$  (12 marks)
78. Solve :  $y(xy + 2x^2 y^2)dx + x(xy - x^2 y^2)dy = 0$  (12 marks)
79. Solve:  $(D^4 - 4D^2 - 5)y = e^x (x + \cos x)$  (15 marks)
80. Reduce the equation  $(px - y)(py + x) = 2p$ , where  $p = \frac{dy}{dx}$  to Clairaut's equation and hence solve it. (15 marks)
81. Solve:  $(x + 2) \frac{d^2 y}{dx^2} - (2x + 5) \frac{dy}{dx} + 2y = (x + 1)e^x$  (15 marks)

82. Solve the following differential equation:  $(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1+x^2)y = x$  (15 marks)

## 2003

83. Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal (12 marks)
84. Solve:  $x\frac{dy}{dx} + y\log y = xy e^x$  (12 marks)
85. Solve:  $(D^5 - D) = 4(e^x + \cos x + x^3)$ , where  $D \equiv \frac{dy}{dx}$ . (15 marks)
86. Solve the differential equation  $(px^2+y^2)(px+y) = (P+I)^2$ , where  $p = \frac{dy}{dx}$ , by reducing it to Clairaut's form using suitable substitutions (15 marks)
87. Solve  $(1-x^2)y'' + (1+x)y' + y = \sin 2[\log(1+x)]$  (15 marks)
88. Solve the differential equation  $x^2y'' - 4xy' + 6y = x^4 \sec^2 x$  by variation of parameters. (15 marks)

## 2002

89. Solve:  $x\frac{dy}{dx} + 3y = x^3 y^2$  (12 marks)
90. Find the values of  $\lambda$  for which all solutions of  $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - \lambda y = 0$  tend to zero as  $x \rightarrow \infty$ . (12 marks)
91. Find the value of constant  $\lambda$  such that the following differential equation becomes exact.  $(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$ . Further, for this value of  $\lambda$ , solve the equation. (15 marks)
92. Solve:  $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$  (15 marks)
93. Using the method of variation of parameters, find the solutions of  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$  with  $y(0) = 0$  and  $\left(\frac{dy}{dx}\right)_{x=0} = 0$  (15 marks)
94. Solve:  $(D-1)(D^2-2D+2)y = e^x$  where  $D = \frac{dy}{dx}$  (15 marks)

## 2001

95. A continuous function  $y(t)$  satisfies the differential equation  $\frac{dy}{dx} = \begin{cases} 1+e^{1-t} & 0 \leq t < 1 \\ 2+2t-3t^2, & 1 \leq t < 5 \end{cases}$  if  $y(0) = -e$  find  $y(2)$  (12 marks)



96. Solve :  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log_e x$  (12 marks)
97. Solve:  $\frac{dy}{dx} + \frac{y}{x} \log_e y = \frac{y(\log_e y)^2}{x^2}$  (15 marks)
98. Find the general solution of  $ayp^2 + (2x-b)p - y = 0$ ,  $a > 0$  (15 marks)
99. Solve:  $(D^2+1)^2y = 24x \cos x$  given that  $y = Dy = D^2y = 0$  and  $D^3y = 12$  when  $x = 0$  (15 marks)
100. Using the method of variation of parameters, solve  $\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$  (15 marks)

## 2000

101. Show that  $3 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} - 8y = 0$  has an integral which is polynomial in x. Deduce the general solution. (12 marks)
102. Reduce  $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R$ , where P, Q, R are functions of x, to the normal form. Hence solve  $\frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$  (15 marks)
103. Solve the differential equation  $y = x - 2ap + ap^2$ . Find the singular solution and interpret it geometrically (15 marks)
104. Show that  $(4x+3y+1)dx + (3x+2y+1)dy = 0$  represents a family of hyperbolas with a common axis and tangent at the vertex (15 marks)
105. Solve  $x \frac{dy}{dx} - y = (x-1) \left( \frac{d^2y}{dx^2} - x + 1 \right)$  by the method of parameters (15 marks)

## 1999

106. Solve the differential equation  $\frac{xdx + ydy}{xdy - ydx} = \left( \frac{1-x^2-y^2}{x^2+y^2} \right)^{1/2}$  (20 marks)
107. Solve  $\frac{d^3y}{dx^3} - 3 \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} - 2y = e^x + \cos x$  (20 marks)
108. By the method of variation parameters solve the differential equation  $\frac{d^2y}{dx^2} + a^2y = \sec(ax)$  (20 marks)

## 1998

109. Solve the differential equation :  $xy - \left( \frac{dy}{dx} \right) = y^3 e^{-x^2}$  (20 marks)

110. Show that the equation :  $(4x+3y+1)dx+(3x+2y+1)dy=0$  represents a family of hyperbolas having as asymptotes the lines  $x+y=0, 2x+y+1=0$ . **(20 marks)**
111. Solve the differential equation:  $y=3px+4p^2$  **(20 marks)**
112. Solve the differential equation:  $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{4x}(x^2+9)$  **(20 marks)**
113. Solve the differential equation:  $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\sin x$  **(20 marks)**
114. Solve the differential equation:  $x^3-\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$  **(20 marks)**

## 1997

115. Solve the initial value problem  $\frac{dy}{dx}=\frac{x}{x^2y+y^3}, y(0)=0$  **(20 marks)**
116. Solve  $(x^2-y^2+3x-y)dx+(x^2-y^2+x-3y)dy=0$  **(20 marks)**
117. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm. find an expression for the radius of the rain drop at anytime. **(20 marks)**
118. Solve  $\frac{d^4y}{dx^4}+6\frac{d^3y}{dx^3}+11\frac{d^2y}{dx^2}+6\frac{dy}{dx}=20e^{-2x}\sin x$  **(20 marks)**
119. Make use of transformation  $y(x)=u(x)\sec x$  to obtain the solution of  $y''-2y'\tan x+5y=0, y(0)=0, y'(0)=\sqrt{6}$  **(20 marks)**
120. Solve  $(1+2x)^2\left(3\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y\right)=8(1+2x)^2, y(0)=0, y'(0)=2$  **(20 marks)**

## 1996

121. find the curves for which the sum of the reciprocals of the radius vector and polar sub tangent is constant. **(20 marks)**
122. Solve :  $x^2(y-px)=yp^2, p=\frac{dy}{dx}$  **(20 marks)**
123. Solve :  $y\sin 2x dx(1+y^2+\cos^2 x)dy=0$  **(20 marks)**
124.  $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$ . Find the value of  $y$  when  $x=\frac{\pi}{2}$ , if it is given that  $y=3$  and  $\frac{dy}{dx}=0$  when  $x=0$  **(20 marks)**
125. Solve :  $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}=x^2+3e^{2x}+4\sin x$  **(20 marks)**
126. Solve:  $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$  **(20 marks)**

## 1995

127. Determine a family of curves for which the ratio of the y - intercept of the tangent to the radius vector is constant (20 marks)
128. Solve  $(2x^2+3y^2-7)xdx+(3x^2+2y^2-8)ydy=0$  (20 marks)
129. Test whether the equation  $(x+y)^2dx-(y^2-2xy-x^2)dy=0$  is exact and hence solve it. (20 marks)
130. Solve  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left( x + \frac{1}{x} \right)$  (20 marks)
131. Determine all real valued solutions of the equations  $y''' - iy'' + y' - iy = 0$ ,  $y' = \frac{dy}{dx}$  (20 marks)
132. Find the solution of the equation  $\frac{d^2y}{dx^2} + 4y = 8 \cos 2x$ , given that  $y = 0$  and  $y' = 2$  when  $x = 0$  (20 marks)

## 1994

133. Solve:  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$  (20 marks)
134. Show that if  $\frac{1}{Q} \left( \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)$  is a function of x only, say, f(x), then  $F(x) = e^{\int f(x) dx}$  is an integration factor of  $Pdx + Qdy = 0$  (20 marks)
135. Find the family of curves whose tangent from an angle  $\frac{\pi}{4}$  with the hyperbola  $xy = c$  (20 marks)
136. Transform the differential equation  $\frac{d^2y}{dx^2} \cos x + \frac{dy}{dx} \sin x - 2y \cos^3 x = 2 \cos^5 x$  into one having z as independent variable where  $z = \sin x$  and solve it. (20 marks)
137. If  $\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$  (a, b and g being positive constants) and  $x = a'$  and  $\frac{dx}{dt} = 0$  when  $t=0$ , show that  $x = a + (a' - a) \cos t \sqrt{\frac{g}{b}}$  (20 marks)
138. Solve  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$  where  $D = \frac{dy}{dx}$  (20 marks)

## 1993

139. Determine the curve for which the radius of curvature is proportional to the slope of the tangent. (20 marks)

140. Show that the system of co focal conics  $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$  is self orthogonal. **(20 marks)**
141. Solve  $\left\{ y \left( 1 + \frac{1}{x} \right) + \cos y \right\} dx + (x + \log x - x \sin y) dy = 0$  **(20 marks)**
142. Solve  $y \frac{d^2 y}{dx^2} - 2 \left( \frac{dy}{dx} \right)^2 = y^2$  **(20 marks)**
143. Solve  $\frac{d^2 y}{dx^2} + \omega_0^2 y = a \cos \omega t$  and discuss the nature of solution as  $\omega \xrightarrow{dt^2} \omega_0$  **(20 marks)**
144. Solve  $(D^4 + D^2 + 1)y = e^{-x/2} \cos \left( x \frac{\sqrt{3}}{2} \right)$  **(20 marks)**

## 1992

145. By eliminating the constants a, b obtain the differential equation for which  $xy = ae^x + be^{-x} + x^2$  is a solution **(20 marks)**
146. Find the orthogonal trajectory of the family of semi cubical parabolas  $ay^2 = x^3$ , where a is a variable parameter. **(20 marks)**
147. Show that  $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$  represents hyperbolas having the following lines as asymptotes  $x + y = 0$ ,  $2x + y + 1 = 0$  **(20 marks)**
148. Solve the following differential equation  $y(1 + xy)dx + x(1 - xy)dy = 0$  **(20 marks)**
149. Find the curves for which the portion of y-axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact. **(20 marks)**
150. Solve the following differential equation:  $(D^2 + 4)y = \sin 2x$ , given that when  $x = 0$ , then  $y = 0$  and  $\frac{dy}{dx} = 2$  **(20 marks)**
151. Solve:  $(D^3 - 1)y = xe^x + \cos^2 x$  **(20 marks)**
152. Solve:  $(x^2 D^2 + xD - 4)y = x^2$  **(20 marks)**